Probing the electromagnetic dipole moments of the tau-neutrino in the $U(1)_{B-L}$ model at the ILC and CLIC energies

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In this work we study the sensitivity on the anomalous magnetic and electric dipole moments of the tau neutrino in the framework of the $SU(2)_L \times U(1)_Y \times U(1)_{B-L}$ electroweak model at future e^+e^- linear colliders, such as the ILC and CLIC. For our study we consider the process $e^+e^- \to (Z, Z', \gamma) \to \nu_\tau \bar{\nu}_\tau \gamma$. For center-of-mass energies of $\sqrt{s} = 1000$–3000 GeV and integrated luminosities of $\mathcal{L} = 500$–2000 fb^{-1}, we derive 95% C.L. limits on the dipole moments $|\mu_{\nu_\tau}(\mu_B)| \leq 6.28 \times 10^{-9}$ and $|d_{\nu_\tau}(\text{ecm})| \leq 1.21 \times 10^{-21}$, which improve on the existing limits by 2 to 3 orders of magnitude. Our study complements other studies on the dipole moments of the tau neutrino at hadron and e^+e^- colliders.

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I. INTRODUCTION

In the Standard Model (SM) [1–3] minimally extended with Dirac neutrino masses, the neutrino magnetic moment induced by radiative corrections is unobservably small [4–6],

$$\mu_{\nu_i} = \frac{3m_e G_F}{4\sqrt{2}\pi^2} m_{\nu_i} \simeq 3.1 \times 10^{-19} \left(\frac{m_{\nu_i}}{1 \text{ eV}}\right) \mu_B, \qquad (1)$$

where $\mu_B = e/2m_e$ is the Bohr magneton. Current limits on these magnetic moments are several orders of magnitude larger, so that a magnetic moment close to these limits would indicate a window for probing effects induced by new physics beyond the SM [6]. Similarly, a neutrino electric dipole moment will also point to new physics and will be of relevance in astrophysics and cosmology, as well as terrestrial neutrino experiments [7]. Some bounds on the neutrino magnetic moment are shown in Table I.

In the case of the anomalous magnetic moment of the tau neutrino, the current best limit on μ_{ν_τ} has been obtained in the Borexino experiment which explores solar neutrinos. Searches for the magnetic moment of the tau neutrino have also been performed in accelerator experiments. The experiment E872 (DONUT) is based on $\nu_\tau e^-$, $\bar{\nu}_\tau e^-$ elastic scattering. In the CERN experiment WA-066, a limit on μ_{ν_τ} was obtained on an assumed flux of tau neutrinos in the neutrino beam. The L3 Collaboration obtained a limit on the magnetic moment of the tau neutrino from a sample of e^+e^- annihilation events at the Z resonance. Experimental limits on the magnetic moment of the tau neutrino are shown in Table II. Others limits on the magnetic moment of μ_{ν_τ} have been reported in the literature [14–36].

The discovery of CP violation in the decays of neutral kaons [40], and later in the decays of neutral B mesons [41] and D^0 [42], shed light on the nature and origin of the violation of this symmetry. CP violation is one of the open problems of the SM. For this reason, the measurement of large amounts of CP violation can be indicative of signs of new physics. The signs of new physics can be analyzed by investigating the electromagnetic dipole moments of the tau neutrino, such as its magnetic moment (MM) and electric dipole moment (EDM), which are defined as a source of CP violation.

Some theoretical limits of the electric dipole moment of the tau neutrino are presented in Table III. Others limits on d_{ν_τ} have been reported in the literature [19–27,29,30].

The $U(1)_{B-L}$ model [45–49] is one of the simplest extensions of the SM with an extra $U(1)$ local gauge symmetry [50], where B and L represent the baryon number and lepton number, respectively. This $B-L$ symmetry plays an important role in various physics scenarios beyond the SM. The features that distinguish the $U(1)_{B-L}$ models from other models are the following. (a) The gauge $U(1)_{B-L}$ symmetry group is contained in the grand unification theory described by a $SO(10)$ group [45]. (b) The scale of the $B-L$ symmetry breaking is related to the mass scale of the heavy right-handed Majorana neutrino mass terms and provides the well-known seesaw mechanism [51–55] to explain light left-handed neutrino mass. (c) The $B-L$ symmetry and the scale of its breaking are tightly connected to the baryogenesis mechanism through leptogenesis [6]. (d) Another distinctive feature of the $U(1)_{B-L}$ models is the possibility of the Z' heavy boson decaying into pairs of heavy neutrinos $\Gamma(Z' \to \nu_h \bar{\nu}_h)$. The

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TABLE I. Bounds on the neutrino magnetic moment.

Experiment/Method	Limit	C.L.	Reference
Laboratory experiment Borexino	$\mu_\nu \leq 5.4 \times 10^{-11} \mu_B$	90%	[8]
Laboratory experiment TEXONO	$\mu_\nu < 2.9 \times 10^{-11} \mu_B$	90%	[9]
Cooling rates of white dwarfs	$\mu_\nu \lesssim 10^{-11} \mu_B$	90%	[10]
Cooling rates of red giants	$\mu_\nu \lesssim 3 \times 10^{-12} \mu_B$	90%	[11]
Supernova energy loss	$\mu_\nu \lesssim (1.1 - 2.7) \times 10^{-12} \mu_B$	90%	[12]
Absence of high-energy events in the SN1987A neutrino signal	$\mu_\nu \lesssim 10^{-12} \mu_B$	90%	[13]
Standard model (Dirac mass)	$\mu_\nu \simeq 3.1 \times 10^{-19} (m_\nu/1\ \mathrm{eV}) \mu_B$		[4–6]

TABLE II. Experimental limits on the magnetic moment of the tau neutrino.

Experiment	Method	Limit	C.L.	Reference
Borexino	Solar neutrino	$\mu_{\nu_\tau} < 1.9 \times 10^{-10} \mu_B$	90%	[8]
E872 (DONUT)	Accelerator $\nu_\tau e^-$, $\bar{\nu}_\tau e^-$	$\mu_{\nu_\tau} < 3.9 \times 10^{-7} \mu_B$	90%	[37]
CERN-WA-066	Accelerator	$\mu_{\nu_\tau} < 5.4 \times 10^{-7} \mu_B$	90%	[38]
L3	Accelerator	$\mu_{\nu_\tau} < 3.3 \times 10^{-6} \mu_B$	90%	[39]

TABLE III. Theoretical limits on the electric dipole moment of the electron neutrino, muon neutrino, and tau neutrino.

Particle	Model	Limit	C.L.	Reference
$\nu_{e,\mu}$	Model independent	$d_{\nu_e,\nu_\mu} < 2 \times 10^{-21}$ ecm	95%	[43]
ν_τ	Effective Lagrangian approach	$d_{\nu_\tau} < 5.2 \times 10^{-17}$ ecm	95%	[15]
ν_τ	Model independent	$d_{\nu_\tau} < O(2 \times 10^{-17}$ ecm$)$	95%	[18]
ν_τ	Vector-like multiplets	$d_{\nu_\tau} < O(10^{-18} - 10^{-20}$ ecm$)$	95%	[44]

model contains an extra gauge boson Z' corresponding to $B-L$ gauge symmetry and an extra SM singlet scalar (heavy Higgs boson H). These new particles can change the SM phenomenology significantly and lead to interesting signatures at current and future colliders, such as the Large Hadron Collider (LHC) [57,58], International Linear Collider (ILC) [59–64], and Compact Linear Collider (CLIC) [65–67].

The $B-L$ model [68,69] is attractive due to its relatively simple theoretical structure. The crucial test of the model is the detection of the new heavy neutral (Z') gauge boson and the new Higgs boson (H). On the other hand, searches for both the heavy gauge boson (Z') and the additional heavy neutral Higgs boson (H) predicted by the $B-L$ model are presently being conducted at the LHC. In this regard, the additional boson Z' of the $B-L$ model has a mass which is given by the relation $M_{Z'} = 2v'g'_1$ [48,49,68,69]. This boson Z' interacts with leptons, quarks, heavy neutrinos, and light neutrinos with interaction strengths proportional to the $B-L$ gauge coupling g'_1. The sensitivity limits on the mass $M_{Z'}$ of the boson Z' of the $U(1)_{B-L}$ model derived by the ATLAS and CMS collaborations are of the order of $\mathcal{O}(1.83 - 2.65)$ TeV [70–78]. It is noteworthy that future LHC runs at 13–14 TeV could increase the Z' mass bounds to higher values, or evidence may be found of its existence. Precision studies of the Z' properties will require a new linear collider [79], which will allow us to perform precision studies of the Higgs sector. We refer the reader to Refs. [48,49,68,69,80–85] for a detailed description of the $B-L$ model.

Our aim in the present paper is to analyze the reaction $e^+e^- \to \nu_\tau \bar{\nu}_\tau \gamma$ in the framework of the $U(1)_{B-L}$ model, and we attribute an anomalous magnetic moment and an electric dipole moment to a massive tau neutrino. It is worth mentioning that at higher s, the dominant contribution involves the exchange of the Z, Z' bosons. The dependence on the magnetic moment (μ_{ν_τ}) and the electric dipole moment (d_{ν_τ}) comes from the radiation of the photon observed by the neutrino or antineutrino in the final state. However, in order to improve the limits on the magnetic moment and the electric dipole moment of the tau neutrino, in our calculation of the process $e^+e^- \to \nu_\tau \bar{\nu}_\tau \gamma$ we consider the contribution that involves the exchange of a virtual photon. In this case, the dependence on the dipole moments comes from a direct coupling to the virtual photon, and the observed photon is a result of initial-state bremsstrahlung.

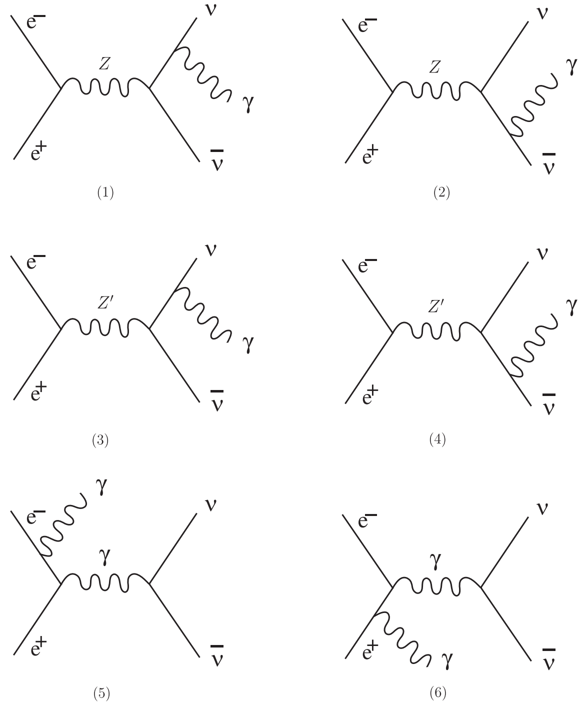

FIG. 1. The Feynman diagrams contributing to the process $e^+e^- \to \nu_\tau\bar{\nu}_\tau\gamma$ (diagrams 1–4) when the $Z(Z')$ vector bosons are produced on mass shell and (diagrams 5 and 6) contributions from anomalous neutrino electromagnetic couplings with initial-state radiation.

The Feynman diagrams which give the most important contributions to the cross section are shown in Fig. 1. This process sets limits on the tau neutrino MM and EDM. In this paper, we take advantage of this fact to set limits on μ_{ν_τ} and d_{ν_τ} for integrated luminosities of 500–2000 fb^{-1} and center-of-mass energies between 1000–3000 GeV, that is to say, in the next generation of linear colliders, namely, the ILC [59] and the CLIC [65].

The L3 Collaboration [39] evaluated the selection efficiency using detector-simulated $e^+e^- \to \nu\bar{\nu}\gamma(\gamma)$ events, random trigger events, and large-angle $e^+e^- \to e^+e^-$ events. From Fig. 1 of Ref. [39], the process $e^+e^- \to \nu\bar{\nu}\gamma$ with γ emitted in the initial state is the sole background in the [44.5°,135.5°] angular range (white histogram). From the same figure, in this angular interval (that is, $-0.7 < \cos\theta_\gamma < 0.7$) we see that only six events were found, (this is the real background), not 14 events. In this case a simple method [24,86,87] is that at the 1σ level (68% C.L.) for a null signal the number of observed events should not exceed the fluctuation of the estimated background events: $N = N_B + \sqrt{N_B}$. Of course, this method is good only when N_B is sufficiently large (i.e., when the Poisson distribution can be approximated with a Gaussian [24,86,87]), but for $N_B > 10$ it is a good approximation. This means that at the 1σ, 2σ, and 3σ levels (68%, 90%, and 95% C.L.) the limits on the nonstandard parameters are found by replacing the equation for the total number of events expected $N = N_B + \sqrt{N_B}$ in the expression $N = \sigma(\mu_{\nu_\tau}, d_{\nu_\tau})\mathcal{L}$. The distributions of the photon energy and the cosine of its polar angle are consistent with SM predictions.

This paper is organized as follows. In Sec, II we present the $B-L$ theoretical model. In Sec. III we present the calculation of the process $e^+e^- \to \nu_\tau\bar{\nu}_\tau\gamma$ in the context of the $B-L$ model. Finally, we present our results and conclusions in Sec. IV.

II. BRIEF REVIEW OF THE $B-L$ THEORETICAL MODEL

Solid evidence for the nonvanishing neutrino masses has been confirmed by various neutrino oscillation phenomena and indicates the evidence of new physics beyond the SM. In the SM, neutrinos are massless due to the absence of right-handed neutrinos and the exact $B-L$ conservation. The most attractive idea to naturally explain the tiny neutrino masses is the seesaw mechanism [52–54,88], in which the right-handed (RH) neutrino singlet under the SM gauge group is introduced. The gauged $U(1)_{B-L}$ model, based on the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$ [51,89], is an elegant and simple extension of the SM in which the RH heavy neutrinos are essential both for anomaly cancellation and preserving gauge invariance. In addition, the mass of RH neutrinos is associated with the $U(1)_{B-L}$ gauge symmetry breaking. Therefore, the fact that neutrinos are massive indicates that the SM requires an extension.

We consider a $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$ model, which is one of the simplest extensions of the SM [48,49,51,68,80–85,89], where $U(1)_{B-L}$ represents the additional gauge symmetry. The gauge-invariant Lagrangian of this model is given by

$$\mathcal{L} = \mathcal{L}_s + \mathcal{L}_{\mathrm{YM}} + \mathcal{L}_f + \mathcal{L}_Y, \tag{2}$$

where $\mathcal{L}_s$, $\mathcal{L}_{\mathrm{YM}}$, $\mathcal{L}_f$, and $\mathcal{L}_Y$ are the scalar, Yang-Mills, fermion, and Yukawa sectors, respectively.

The model consists of one doublet Φ and one singlet χ, and we briefly describe the Lagrangian including the scalar, fermion, and gauge sectors, respectively. The Lagrangian for the gauge sector is given by [48,84,90,91],

$$\mathcal{L}_g = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}Z'_{\mu\nu}Z'^{\mu\nu}, \tag{3}$$

where $W^a_{\mu\nu}$, $B_{\mu\nu}$, and $Z'_{\mu\nu}$ are the field-strength tensors for $SU(2)_L$, $U(1)_Y$, and $U(1)_{B-L}$, respectively.

The Lagrangian for the scalar sector of the model is

$$\mathcal{L}_s = (D^\mu\Phi)^\dagger(D_\mu\Phi) + (D^\mu\chi)^\dagger(D_\mu\chi) - V(\Phi,\chi), \tag{4}$$

where the potential term is [82]

$$V(\Phi,\chi) = m^2(\Phi^\dagger\Phi) + \mu^2|\chi|^2 + \lambda_1(\Phi^\dagger\Phi)^2 + \lambda_2|\chi|^4 + \lambda_3(\Phi^\dagger\Phi)|\chi|^2, \quad (5)$$

where Φ and χ are the complex scalar Higgs doublet and singlet fields, respectively. The covariant derivative is given by [80–82]

$$D_\mu = \partial_\mu + ig_s t^\alpha G^\alpha_\mu + i[gT^a W^a_\mu + g_1 Y B_\mu + (\tilde{g}Y + g'_1 Y_{B-L})B'_\mu], \quad (6)$$

where g_s, g, g_1, and g'_1 are the $SU(3)_C$, $SU(2)_L$, $U(1)_Y$, and $U(1)_{B-L}$ couplings with t^α, T^a, Y, and Y_{B-L} being their respective group generators. The mixing between the two Abelian groups is described by the new coupling $\tilde{g}$. The electromagnetic charges on the fields are the same as those of the SM, and the Y_{B-L} charges for quarks, leptons, and the scalar fields are given by $Y^{\rm quarks}_{B-L} = 1/3$, $Y^{\rm leptons}_{B-L} = -1$ (with no distinction between generations to ensure universality), $Y_{B-L}(\Phi) = 0$, and $Y_{B-L}(\chi) = 2$ [48,49,80–82] (to preserve the gauge invariance of the model), respectively.

An effective coupling and effective charge such as g' and Y' are usually introduced as $g'Y' = \tilde{g}Y + g'_1 Y_{B-L}$, and some specific benchmark models [92,93] can be recovered with particular choices of both $\tilde{g}$ and g'_1 gauge couplings at a given scale, generally the electroweak scale. For instance, the pure $B-L$ model is obtained from the condition $\tilde{g} = 0$ ($Y' = Y_{B-L}$), which implies the absence of mixing at the electroweak scale. Other benchmark models of the general parametrization are the sequential Standard Model (SSM), the $U(1)_R$ model, and the $U(1)_\chi$ model. The SSM is reproduced from the condition $g'_1 = 0$ ($Y' = Y$), and the $U(1)_R$ extension is realized from the condition $\tilde{g} = -2g'_1$, while the $SO(10)$-inspired $U(1)_\chi$ model is described by $\tilde{g} = -\frac{4}{5}g'_1$.

The doublet and singlet scalars are

$$\Phi = \begin{pmatrix} G^\pm \\ \frac{v+\phi^0+iG_Z}{\sqrt{2}} \end{pmatrix}, \qquad \chi = \left(\frac{v' + \phi'^0 + iz'}{\sqrt{2}}\right), \quad (7)$$

where $G^\pm$, G_Z, and z' the Goldstone bosons of $W^\pm$, Z, and Z', respectively, while $v \approx 246$ GeV is the electroweak symmetry breaking scale and v' is the B-L symmetry breaking scale constrained by the electroweak precision measurement data, whose value is assumed to be of the order TeV.

After spontaneous symmetry breaking, the two scalar fields can be written as

$$\Phi = \begin{pmatrix} 0 \\ \frac{v+\phi^0}{\sqrt{2}} \end{pmatrix}, \qquad \chi = \frac{v' + \phi'^0}{\sqrt{2}}, \quad (8)$$

where v and v' are real and positive.

In Table IV, the interactions of h and H with the gauge bosons and scalar are expressed in terms of the parameters of the $B-L$ model.

To determine the mass spectrum of the gauge bosons we have to expand the scalar kinetic terms, as in the SM. We expect that there exists a massless gauge boson (the photon), while the other gauge bosons become massive. The extension we are studying is in the Abelian sector of the SM gauge group, so that the charged gauge bosons $W^\pm$ will have masses given by their SM expressions related to the $SU(2)_L$ factor only. The other gauge boson masses are not so simple to identify because of mixing. In fact, analogously to the SM, the fields of definite mass are linear combinations of B^μ, W^μ_3, and B'^μ. The relation between the neutral gauge bosons (B^μ, W^μ_3, and B'^μ) and the corresponding mass eigenstates are given by [68,69,80,81]

TABLE IV. The new couplings of the Z, Z' bosons with the SM fermions and vector-boson and scalar coupling in the $B-L$ model. $g = e/\sin\theta_W$ and θ_{B-L} is the $Z-Z'$ mixing angle.

Particle	Couplings
$f\bar{f}Z$	$g^f_V = T^f_3\cos\theta_{B-L} - 2Q_f\sin^2\theta_W\cos\theta_{B-L} + \frac{2g'_1}{g}\cos\theta_W\sin\theta_{B-L}$,
	$g^f_A = T^f_3\cos\theta_{B-L}$
$f\bar{f}Z'$	$g'^f_V = -T^f_3\sin\theta_{B-L} - 2Q_f\sin^2\theta_W\sin\theta_{B-L} + \frac{2g'_1}{g}\cos\theta_W\cos\theta_{B-L}$,
	$g'^f_A = -T^f_3\sin\theta_{B-L}$
$Z_\mu Z'_\nu h$	$g_{ZZ'h} = 2i[\frac{1}{4}v\cos\alpha f(\theta_{B-L},g'_1) - v'\sin\alpha g(\theta_{B-L},g'_1)]g_{\mu\nu}$,
	$f(\theta_{B-L},g'_1) = -\sin(2\theta')(g_1^2 + g_2^2 + g'^2_1) - 2\cos(2\theta')g_1\sqrt{g_1^2+g_2^2}$,
	$g(\theta_{B-L},g'_1) = \frac{1}{4}\sin(2\theta')g'^2_1$
$Z_\mu Z'_\nu H$	$g_{ZZ'H} = 2i[\frac{1}{4}v\sin\alpha f(\theta_{B-L},g'_1) + v'\cos\alpha g(\theta_{B-L},g'_1)]g_{\mu\nu}$
$W^-_\mu(p_1)W^+_\nu(p_2)Z'_\rho(p_3)$	$g_{W^-W^+Z'} = -ig\cos\theta_W\sin\theta_{B-L}[(p_1-p_2)_\rho g_{\mu\nu} + (p_2-p_3)_\mu g_{\nu\rho} + (p_3-p_1)_\nu g_{\rho\nu}]$

$$\begin{pmatrix} B^\mu \\ W^{3\mu} \\ B'^\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W\cos\theta_{B-L} & \sin\theta_W\sin\theta_{B-L} \\ \sin\theta_W & \cos\theta_W\cos\theta_{B-L} & -\cos\theta_W\sin\theta_{B-L} \\ 0 & \sin\theta_{B-L} & \cos\theta_{B-L} \end{pmatrix} \begin{pmatrix} A^\mu \\ Z^\mu \\ Z'^\mu \end{pmatrix}, \tag{9}$$

with $-\frac{\pi}{4} \leq \theta_{B-L} \leq \frac{\pi}{4}$, such that

$$\tan 2\theta_{B-L} = \frac{2\tilde{g}\sqrt{g^2+g_1^2}}{\tilde{g}^2 + 16(\frac{v'}{v})^2 g_1'^2 - g^2 - g_1^2}, \tag{10}$$

and the mass spectrum of the gauge bosons is given by

$$\begin{aligned} M_\gamma &= 0, \\ M_{W^\pm} &= \frac{1}{2}vg, \\ M_Z &= \frac{v}{2}\sqrt{g^2+g_1^2}\sqrt{\frac{1}{2}\left(\frac{\tilde{g}^2+16(\frac{v'}{v})^2 g_1'^2}{g^2+g_1^2}+1\right) - \frac{\tilde{g}}{\sin 2\theta_{B-L}\sqrt{g^2+g_1^2}}}, \\ M_{Z'} &= \frac{v}{2}\sqrt{g^2+g_1^2}\sqrt{\frac{1}{2}\left(\frac{\tilde{g}^2+16(\frac{v'}{v})^2 g_1'^2}{g^2+g_1^2}+1\right) + \frac{\tilde{g}}{\sin 2\theta_{B-L}\sqrt{g^2+g_1^2}}}, \end{aligned} \tag{11}$$

where M_Z and $M_{W^\pm}$ are the SM gauge boson masses and $M_{Z'}$ is the mass of the new neutral gauge boson Z', which strongly depends on v' and g_1'. For $\tilde{g} = 0$, there is no mixing between the new and SM gauge bosons Z' and Z. In this case, the $U(1)_{B-L}$ model is called the pure or minimal $U(1)_{B-L}$ model. In this article we consider the case $\tilde{g} \neq 0$, which is mostly determined by the other gauge couplings g_1 and g_1' [94–96]. The electroweak precision measurement data can give stringent constraints on the $Z - Z'$ mixing angle θ_{B-L} expressed in Eq. (10) [97].

In the Lagrangian of the $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$ model, the terms for the interactions between neutral gauge bosons Z, Z' and a pair of fermions of the SM can be written in the form [33,48,49,98–100]

$$\begin{aligned} \mathcal{L}_{NC} = &\frac{-ig}{\cos\theta_W}\sum_f \bar{f}\gamma^\mu \frac{1}{2}(g_V^f - g_A^f\gamma^5) f Z_\mu \\ &+ \frac{-ig}{\cos\theta_W}\sum_f \bar{f}\gamma^\mu \frac{1}{2}(g_V'^f - g_A'^f\gamma^5) f Z'_\mu. \end{aligned} \tag{12}$$

From this Lagrangian we determine the expressions for the new couplings of the Z, Z' bosons with the SM fermions, which are given in Table IV. The couplings $g_V^f(g_V'^f)$ and $g_A^f(g_A'^f)$ depend on the $Z - Z'$ mixing angle θ_{B-L} and the coupling constant g_1' of the $B - L$ interaction. In these couplings, the current bound on the mixing angle is $|\theta_{B-L}| \leq 10^{-3}$ [24]. In the decoupling limit, when $\theta_{B-L} = 0$ and $g_1' = 0$, the couplings of the SM are recovered.

III. THE DECAY WIDTHS OF THE Z' BOSON IN THE $B-L$ MODEL

In this section we present the decay widths of the Z' boson [98,99,101–105] in the context of the $B - L$ model needed in the calculation of the cross section for the process $e^+e^- \to \nu_\tau\bar{\nu}_\tau\gamma$. The decay width of the Z' boson to fermions is given by

$$\Gamma(Z' \to f\bar{f}) = \frac{2G_F}{3\pi\sqrt{2}} N_c M_Z^2 M_{Z'}\sqrt{1-\frac{4M_f^2}{M_{Z'}^2}}\left[(g_V'^f)^2\left\{1+2\left(\frac{M_f^2}{M_{Z'}^2}\right)\right\} + (g_A'^f)^2\left\{1-4\left(\frac{M_f^2}{M_{Z'}^2}\right)\right\}\right], \tag{13}$$

where N_c is the color factor ($N_c = 1$ for leptons and $N_c = 3$ for quarks) and the couplings $g_V'^f$ and $g_A'^f$ of the Z' boson with the SM fermions are given in Table IV.

The decay width of the Z' boson to heavy neutrinos is

$$\Gamma(Z' \to \nu_h\bar{\nu}_h) = \frac{g_1'^2}{24\pi}\sin^2\theta_{B-L} M_{Z'}\sqrt{\left(1-\frac{4M_{\nu_h}^2}{M_{Z'}^2}\right)^3}, \tag{14}$$

where the width given by Eq. (14) implies that the right-handed neutrino must be lighter than half the Z' mass, $M_{\nu_h} < \frac{M_{Z'}}{2}$, and the conditions under which this inequality holds is for coupled heavy neutrinos, i.e., with a mass less than $\frac{M_{Z'}}{2}$. The possibility of the Z' heavy boson decaying into pairs of heavy neutrinos is certainty one of its most interesting features.

The Z' partial decay widths involving vector bosons and the scalar bosons are

$$\Gamma(Z' \to W^+W^-) = \frac{G_F M_W^2}{24\pi\sqrt{2}} \cos^2\theta_W \sin^2\theta_{B-L} M_{Z'} \left(\frac{M_{Z'}}{M_Z}\right)^4 \sqrt{\left(1 - 4\frac{M_W^2}{M_{Z'}^2}\right)^3} \left[1 + 20\frac{M_W^2}{M_{Z'}^2} + 12\frac{M_W^4}{M_{Z'}^4}\right], \tag{15}$$

$$\Gamma(Z' \to Zh) = \frac{G_F M_Z^2 M_{Z'}}{24\pi\sqrt{2}} \sqrt{\lambda_h}\left[\lambda_h + 12\frac{M_Z^2}{M_{Z'}^2}\right][f(\theta_{B-L}, g'_1)\cos\alpha + g(\theta_{B-L}, g'_1)\sin\alpha]^2, \tag{16}$$

$$\Gamma(Z' \to ZH) = \frac{G_F M_Z^2 M_{Z'}}{24\pi\sqrt{2}} \sqrt{\lambda_H}\left[\lambda_H + 12\frac{M_Z^2}{M_{Z'}^2}\right][f(\theta_{B-L}, g'_1)\sin\alpha - g(\theta_{B-L}, g'_1)\cos\alpha]^2, \tag{17}$$

where

$$\begin{aligned}
\lambda_{h,H}\left(1, \frac{M_Z^2}{M_{Z'}^2}, \frac{M_{h,H}^2}{M_{Z'}^2}\right) &= 1 + \left(\frac{M_Z^2}{M_{Z'}^2}\right)^2 + \left(\frac{M_{h,H}^2}{M_{Z'}^2}\right)^2 - 2\left(\frac{M_Z^2}{M_{Z'}^2}\right) - 2\left(\frac{M_{h,H}^2}{M_{Z'}^2}\right) - 2\left(\frac{M_Z^2}{M_{Z'}^2}\right)\left(\frac{M_{h,H}^2}{M_{Z'}^2}\right), \\
f(\theta_{B-L}, g'_1) &= \left(1 + \frac{v^2 g_1'^2}{4M_Z^2}\right)\sin(2\theta_{B-L}) + \left(\frac{v g'_1}{M_Z}\right)\cos(2\theta_{B-L}), \\
g(\theta_{B-L}, g'_1) &= \left(\frac{v v'}{4M_Z^2}\right) g_1'^2 \sin(2\theta_{B-L}).
\end{aligned} \tag{18}$$

In the $B-L$ model, the heavy gauge boson mass $M_{Z'}$ satisfies the relation $M_{Z'} = 2v'g'_1$ [48,49,68,69,80,81], and considering the most recent limit of $\frac{M_{Z'}}{g'_1} \geq 6.9$ TeV [93,106,107], it is possible to obtain a direct bound on the $B-L$ breaking scale v'. In our next numerical calculation, we will take $v' = 3.45$ TeV, while $\alpha = \frac{\pi}{9}$ for the $h-H$ mixing angle, in correspondence with Refs. [48,57,58,108].

IV. THE TOTAL CROSS SECTION

In this section we calculate the total cross section for the reaction $e^+e^- \to \nu_\tau\bar{\nu}_\tau\gamma$. The respective transition amplitudes are thus given by

$$\begin{aligned}
\mathcal{M}_1 = &\frac{-g^2}{4\cos^2\theta_W(l^2 - m_\nu^2)}[\bar{u}(p_3)\Gamma^\alpha(\not{l} + m_\nu)\gamma^\beta(g_V^\nu - g_A^\nu\gamma_5)v(p_4)] \\
&\times \frac{(g_{\alpha\beta} - p_\alpha p_\beta/M_Z^2)}{[(p_1+p_2)^2 - M_Z^2 - iM_Z\Gamma_Z]}[\bar{u}(p_2)\gamma^\alpha(g_V^e - g_A^e\gamma_5)v(p_1)]\epsilon_\alpha^\lambda,
\end{aligned} \tag{19}$$

$$\begin{aligned}
\mathcal{M}_2 = &\frac{-g^2}{4\cos^2\theta_W(l'^2 - m_\nu^2)}[\bar{u}(p_3)\gamma^\beta(g_V^\nu - g_A^\nu\gamma_5)(\not{l}' + m_\nu)\Gamma^\alpha v(p_4)] \\
&\times \frac{(g_{\alpha\beta} - p_\alpha p_\beta/M_Z^2)}{[(p_1+p_2)^2 - M_Z^2 - iM_Z\Gamma_Z]}[\bar{u}(p_2)\gamma^\alpha(g_V^e - g_A^e\gamma_5)v(p_1)]\epsilon_\alpha^\lambda,
\end{aligned} \tag{20}$$

$$\begin{aligned}
\mathcal{M}_3 = &\frac{-g^2}{4\cos^2\theta_W(r^2 - m_\nu^2)}[\bar{u}(p_3)\Gamma^\alpha(\not{r} + m_\nu)\gamma^\beta(g_V'^\nu - g_A'^\nu\gamma_5)v(p_4)] \\
&\times \frac{(g_{\alpha\beta} - p_\alpha p_\beta/M_{Z'}^2)}{[(p_1+p_2)^2 - M_{Z'}^2 - iM_{Z'}\Gamma_{Z'}]}[\bar{u}(p_2)\gamma^\alpha(g_V'^e - g_A'^e\gamma_5)v(p_1)]\epsilon_\alpha^\lambda,
\end{aligned} \tag{21}$$

$$\mathcal{M}_4 = \frac{-g^2}{4\cos^2\theta_W(r'^2 - m_\nu^2)}[\bar{u}(p_3)\gamma^\beta(g'^\nu_{\rm v} - g'^\nu_A\gamma_5)(\not{r}' + m_\nu)\Gamma^\alpha v(p_4)] \times \frac{(g_{\alpha\beta} - p_\alpha p_\beta/M_{Z'}^2)}{[(p_1+p_2)^2 - M_{Z'}^2 - iM_{Z'}\Gamma_{Z'}]}[\bar{u}(p_2)\gamma^\alpha(g'^e_{\rm v} - g'^e_A\gamma_5)v(p_1)]\epsilon^\lambda_\alpha, \tag{22}$$

$$\mathcal{M}_5 = \frac{e^2}{(k^2 - m_e^2)}[\bar{u}(p_3)\Gamma^\alpha v(p_4)]\frac{g_{\alpha\beta}}{(p_1+p_2)^2}[\bar{u}(p_2)\gamma^\alpha(\not{k} + m_e)\gamma^\beta v(p_1)]\epsilon^\lambda_\alpha, \tag{23}$$

and

$$\mathcal{M}_6 = \frac{e^2}{(k'^2 - m_e^2)}[\bar{u}(p_3)\Gamma^\alpha v(p_4)]\frac{g_{\alpha\beta}}{(p_1+p_2)^2}[\bar{u}(p_2)\gamma^\beta(\not{k}' + m_e)\gamma^\alpha v(p_1)]\epsilon^\lambda_\alpha, \tag{24}$$

where the most general expression for the tau neutrino electromagnetic vertex consistent with Lorentz and electromagnetic gauge invariance may be parametrized in terms of four form factors:

$$\Gamma^\alpha = eF_1(q^2)\gamma^\alpha + \frac{ie}{2m_{\nu_\tau}}F_2(q^2)\sigma^{\alpha\mu}q_\mu + eF_3(q^2)\gamma_5\sigma^{\alpha\mu}q_\mu + eF_4(q^2)\gamma_5(\gamma^\mu q^2 - \not{q}q^\mu), \tag{25}$$

where e is the charge of the electron, m_{ν_τ} is the mass of the tau neutrino, q^μ is the photon momentum, and $F_{1,2,3,4}(q^2)$ are the electromagnetic form factors of the neutrino, corresponding to the charge radius, MM, EDM, and anapole moment, respectively, at $q^2 = 0$ [15,109–114], while ϵ^λ_α is the polarization vector of the photon. l, $r(k)$ and l', $r'(k')$ stand for the momentum of the virtual neutrino (electron) and antineutrino (positron), respectively. The form factors corresponding to the charge radius and the anapole moment do not concern us here.

The MM and EDM give a contribution to the total cross section for the process $e^+e^- \to \nu_\tau\bar{\nu}_\tau\gamma$ of the form

$$\begin{aligned}\sigma_{\rm Tot}(e^+e^- \to \nu_\tau\bar{\nu}_\tau\gamma) = \int \frac{\alpha^2}{96\pi}(\kappa^2\mu_B^2 + d^2_{\nu_\tau}) &\times \Bigg\{4\left[\frac{(g^e_{\rm v})^2 + (g^e_A)^2}{x_W^2(1-x_W)^2}\right]\left[\frac{((g^\nu_{\rm v})^2 + (g^\nu_A)^2)(s - 2\sqrt{s}E_\gamma) + (g^\nu_A)^2E_\gamma^2\sin^2\theta_\gamma}{(s - M_Z^2)^2 + M_Z^2\Gamma_Z^2}\right]. \\ &+ 4\left[\frac{(g'^e_{\rm v})^2 + (g'^e_A)^2}{x_W^2(1-x_W)^2}\right]\left[\frac{((g'^\nu_{\rm v})^2 + (g'^\nu_A)^2)(s - 2\sqrt{s}E_\gamma) + (g'^\nu_A)^2E_\gamma^2\sin^2\theta_\gamma}{(s - M_{Z'}^2)^2 + M_{Z'}^2\Gamma_{Z'}^2}\right]. \\ &+ 32\left[\frac{s - 2\sqrt{s}E_\gamma + 2E_\gamma^2 - E_\gamma^2\sin^2\theta_\gamma}{sE_\gamma^2\sin^2\theta_\gamma}\right] \\ &+ 6\left[\frac{(g^e_{\rm v}g'^e_{\rm v} + g^e_Ag'^e_A)}{x_W^2(1-x_W)^2}\right]\left[\frac{(s - M_Z^2)(s - M_{Z'}^2) + M_ZM_{Z'}\Gamma_Z\Gamma_{Z'}}{[(s - M_Z^2)^2 + M_Z^2\Gamma_Z^2][(s - M_{Z'}^2)^2 + M_{Z'}^2\Gamma_{Z'}^2]}\right]. \\ &\times [(g^e_{\rm v}g'^e_{\rm v} + g^e_Ag'^e_A)(s - 2\sqrt{s}E_\gamma) + (g^\nu_Ag'^\nu_A)E_\gamma^2\sin^2\theta_\gamma]\Bigg\}E_\gamma dE_\gamma d\cos\theta_\gamma,\end{aligned} \tag{26}$$

where $x_W \equiv \sin^2\theta_W$, and E_γ and $\cos\theta_\gamma$ are the energy and opening angle of the emitted photon.

The expression given in Eq. (26) corresponds to the total cross section with the exchange of the Z, Z', γ bosons. The SM expression for the cross section of the reaction $e^+e^- \to \nu_\tau\bar{\nu}_\tau\gamma$ can be obtained in the decoupling limit when $\theta_{B-L} = 0$, $g'_1 = 0$, and $\alpha = 0$. In this case, the terms that depend on θ_{B-L}, g'_1 and α in Eq. (26) are zero, and Eq. (26) is reduced to the expression given in Ref. [14] for the Standard Model minimally extended to include massive Dirac neutrinos.

V. RESULTS AND CONCLUSIONS

In order to evaluate the integral of the total cross section as a function of the parameters of the model (that is to say, μ_{ν_τ} and d_{ν_τ}), we require cuts on the photon angle and energy to avoid divergences when the integral is evaluated at the important intervals of each experiment. We integrate over θ_γ from 44.5° to 135.5° and E_γ from 15 to 100 GeV. We use the following values for numerical computation [24]: $\sin^2\theta_W = 0.23126 \pm 0.00022$, $m_\tau = 1776.82 \pm 0.16$ MeV, $m_b = 4.6 \pm 0.18$ GeV, $m_t = 172 \pm 0.9$ GeV,

TABLE V. Bounds on the magnetic moment μ_{ν_τ} and electric dipole moment d_{ν_τ} for $\sqrt{s} = 1000, 2000, 3000$ GeV and $\mathcal{L} = 500, 1000, 2000$ fb^{-1} at 1σ, 2σ, and 3σ.

$\mathcal{L} = 500, 1000, 2000$ fb^{-1}		
$\sqrt{s} = 1000$ GeV; $M_{Z'} = 1000$ GeV, $g_1' = 0.145$		
C.L.	$\lvert\mu_{\nu_\tau}(\mu_B)\rvert$	$\lvert d_{\nu_\tau}(\mathrm{ecm})\rvert$
1σ	$(3.11, 2.20, 1.55) \times 10^{-8}$	$(6.01, 4.25, 3.00) \times 10^{-19}$
2σ	$(3.53, 2.50, 1.76) \times 10^{-8}$	$(6.82, 4.82, 3.41) \times 10^{-19}$
3σ	$(3.91, 2.75, 1.95) \times 10^{-8}$	$(7.55, 5.34, 3.77) \times 10^{-19}$
$\sqrt{s} = 2000$ GeV; $M_{Z'} = 2000$ GeV, $g_1' = 0.290$		
C.L.	$\lvert\mu_{\nu_\tau}(\mu_B)\rvert$	$\lvert d_{\nu_\tau}(\mathrm{ecm})\rvert$
1σ	$(1.51, 1.07) \times 10^{-8}$, 7.57×10^{-9}	$(2.92, 2.06, 1.46) \times 10^{-19}$
2σ	$(1.72, 1.21) \times 10^{-8}$, 8.60×10^{-9}	$(3.31, 2.34, 1.65) \times 10^{-19}$
3σ	$(1.90, 1.34) \times 10^{-8}$, 9.52×10^{-9}	$(3.67, 2.59, 1.83) \times 10^{-19}$
$\sqrt{s} = 3000$ GeV; $M_{Z'} = 3000$ GeV, $g_1' = 0.435$		
C.L.	$\lvert\mu_{\nu_\tau}(\mu_B)\rvert$	$\lvert d_{\nu_\tau}(\mathrm{ecm})\rvert$
1σ	1.00×10^{-8}, $(7.07, 5.00) \times 10^{-9}$	$(1.93, 1.36) \times 10^{-19}$, 9.65×10^{-20}
2σ	1.13×10^{-8}, $(8.03, 5.68) \times 10^{-9}$	$(2.19, 1.55, 1.09) \times 10^{-19}$
3σ	1.25×10^{-8}, $(8.89, 6.28) \times 10^{-9}$	$(2.42, 1.71, 2.21) \times 10^{-19}$

$M_{W^\pm} = 80.389 \pm 0.023$ GeV, $M_Z = 91.1876 \pm 0.0021$ GeV, $\Gamma_Z = 2.4952 \pm 0.0023$ GeV, $M_h = 125 \pm 0.4$ GeV, and $M_H = 500$ GeV. Considering the most recent limit from Refs. [93,106,107],

$$\frac{M_{Z'}}{g_1'} \geq 6.9 \text{ TeV}, \tag{27}$$

it is possible to obtain a direct bound on the $B-L$ breaking scale v' and take $v' = 3.45$ TeV and $\alpha = \frac{\pi}{9}$. In our numerical analysis, we obtain the total cross section $\sigma_{\mathrm{Tot}} = \sigma_{\mathrm{Tot}}(\mu_{\nu_\tau}, d_{\nu_\tau}, \sqrt{s}, M_{Z'}, g_1', \theta_{B-L}, \alpha)$. Thus, in our numerical computation, we will assume that $\sqrt{s}$, $M_{Z'}$, g_1', θ_{B-L}, and α are free parameters.

As was discussed in Refs. [14,39,115,116], $N \approx \sigma_{\mathrm{Tot}}(\mu_{\nu_\tau}, d_{\nu_\tau}, \sqrt{s}, M_{Z'}, g_1', \theta_{B-L}, \alpha)\mathcal{L}$, where $N = N_B + \sqrt{N_B}$ is the total number of $e^+e^- \to \nu_\tau\bar{\nu}_\tau\gamma$ events expected at the 1σ, 2σ, and 3σ levels (as mentioned in the Introduction) and $\mathcal{L} = 500 - 2000$ fb^{-1} according to the

TABLE VI. Bounds on the magnetic moment μ_{ν_τ} and electric dipole moment d_{ν_τ} for $\sqrt{s} = 1000, 2000, 3000$ GeV and $\mathcal{L} = 500, 1000, 2000$ fb^{-1} at 1σ, 2σ, and 3σ.

$\mathcal{L} = 500, 1000, 2000$ fb^{-1}		
$\sqrt{s} = 1000$ GeV		
C.L.	$\lvert\mu_{\nu_\tau}(\mu_B)\rvert$	$\lvert d_{\nu_\tau}(\mathrm{ecm})\rvert$
1σ	$(2.22, 1.57, 1.11) \times 10^{-7}$	$(4.29, 3.03, 2.14) \times 10^{-18}$
2σ	$(2.52, 1.78, 1.26) \times 10^{-7}$	$(4.87, 3.44, 2.43) \times 10^{-18}$
3σ	$(2.79, 1.97, 1.39) \times 10^{-7}$	$(5.39, 3.81, 2.69) \times 10^{-18}$
$\sqrt{s} = 2000$ GeV		
C.L.	$\lvert\mu_{\nu_\tau}(\mu_B)\rvert$	$\lvert d_{\nu_\tau}(\mathrm{ecm})\rvert$
1σ	1.08×10^{-7}, $(7.64, 5.40) \times 10^{-8}$	$(2.08, 1.47, 1.04) \times 10^{-18}$
2σ	1.22×10^{-7}, $(8.68, 6.14) \times 10^{-8}$	$(2.36, 1.67, 1.18) \times 10^{-18}$
3σ	1.35×10^{-7}, $(9.61, 6.79) \times 10^{-8}$	$(2.62, 1.85, 1.31) \times 10^{-18}$
$\sqrt{s} = 3000$ GeV		
C.L.	$\lvert\mu_{\nu_\tau}(\mu_B)\rvert$	$\lvert d_{\nu_\tau}(\mathrm{ecm})\rvert$
1σ	$(7.14, 5.05, 3.57) \times 10^{-8}$	1.37×10^{-18}, $(9.74, 6.88) \times 10^{-19}$
2σ	$(8.11, 5.73, 4.05) \times 10^{-8}$	$(1.56, 1.10) \times 10^{-18}$, 7.82×10^{-19}
3σ	$(8.97, 6.34, 4.48) \times 10^{-9}$	$(1.73, 1.22) \times 10^{-18}$, 8.65×10^{-19}

data reported by the ILC and CLIC [59,65]. Taking this into consideration, we can obtain a limit for the tau neutrino magnetic moment with $d_{\nu_\tau} = 0$.

As an indicator of the order of magnitude of the dipole moments, we present the bounds obtained for the magnetic moment μ_{ν_τ} and electric dipole moment d_{ν_τ} in Table V for several center-of-mass energies ($\sqrt{s} = 1000$, 2000, 3000 GeV), integrated luminosities ($\mathcal{L} = 500$, 1000, 2000 fb^{-1}), and heavy gauge boson masses ($M_{Z'} = 1000$, 2000, 3000 GeV) with $g'_1 = 0.145, 0.290, 0.435$ at $1\sigma, 2\sigma$, and 3σ, respectively. It is worth mentioning that the values reported in Table V for the dipole moments are determined while preserving the relationship between $M_{Z'}$ and g'_1 given in Eq. (27). This relationship will always hold throughout the article. We observe that the results obtained in Table V are better than those reported in the literature [14–18,26–34,38,39].

The previous analysis and comments can readily be translated to the EDM of the τ neutrino with $\mu_{\nu_\tau} = 0$. The resulting limits for the EDM as a function of $\sqrt{s}$, $M_{Z'}$, and g'_1 are shown in Table V.

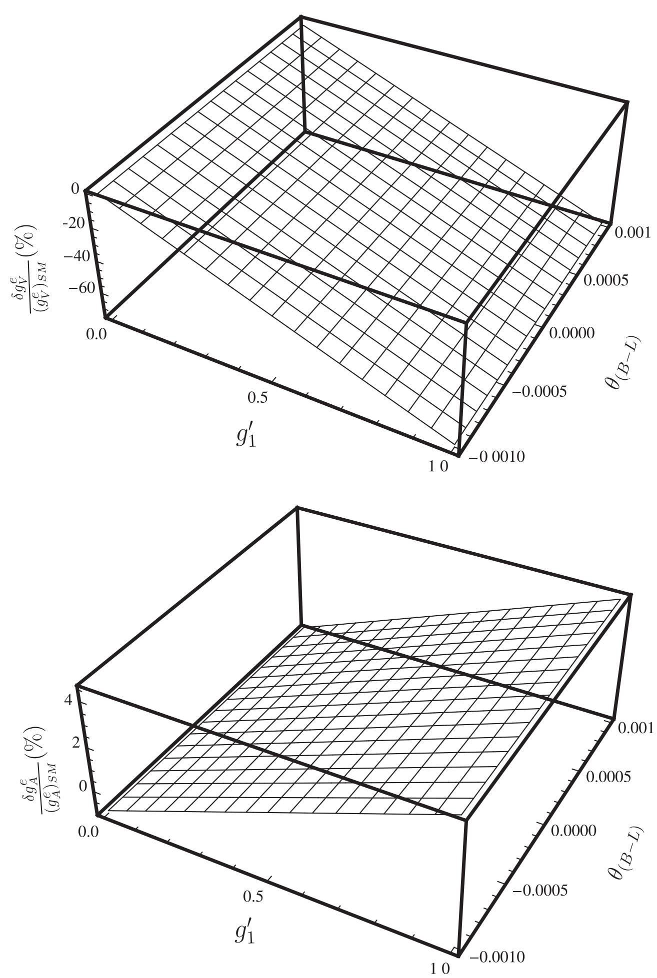

FIG. 2. Top panel: The relative correction $\frac{\delta g_V^e}{(g_V^e)_{\rm SM}}$ as a function of (g'_1, θ_{B-L}). Bottom panel: The relative correction $\frac{\delta g_A^e}{(g_A^e)_{\rm SM}}$ as a function of (g'_1, θ_{B-L}).

In the case of the minimally extended SM [14], i.e., in the decoupling limit when $\theta_{B-L} = 0$, $g'_1 = 0$, and $\alpha = 0$, the bounds generated for the dipole moments are given in Table VI. These bounds are weaker than those obtained with the $U(1)_{B-L}$ model.

The vector and axial-vector e^+e^-Z couplings g_V^e and g_A^e, which depend on g'_1 and θ_{B-L}, are given in Table IV. To see the dependence of g_V^e and g_A^e on the parameters of the model, we plot the relative corrections $\frac{\delta g_V^e}{(g_V^e)_{\rm SM}} = \frac{(g_V^e)_{B-L} - (g_V^e)_{\rm SM}}{(g_V^e)_{\rm SM}}$ and $\frac{\delta g_A^e}{(g_A^e)_{\rm SM}} = \frac{(g_A^e)_{B-L} - (g_A^e)_{\rm SM}}{(g_A^e)_{\rm SM}}$ as functions of (g'_1, θ_{B-L}) in Fig. 2. From the top panel, we can see that the absolute value of the relative correction $\frac{\delta g_V^e}{(g_V^e)_{\rm SM}}$ increases when the parameter g'_1 increases and is almost independent of the mixing angle θ_{B-L}. However, the absolute value of $\frac{\delta g_V^e}{(g_V^e)_{\rm SM}}$ is in the range 10–70% in most of the parameter space. In the

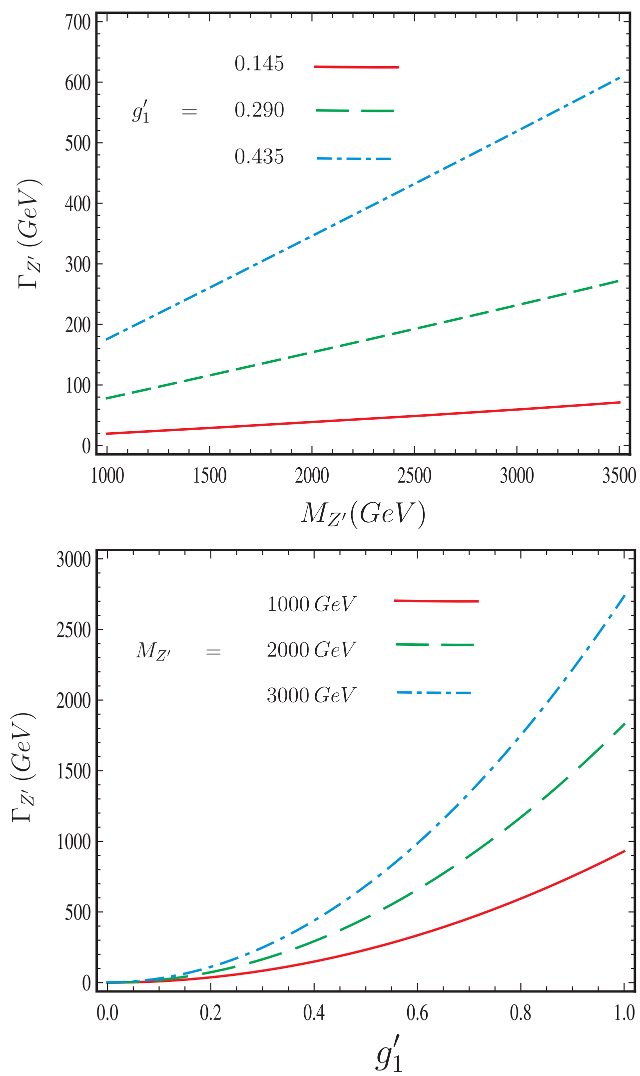

FIG. 3. Top panel: The Z' width as a function of $M_{Z'}$ for fixed values of g'_1. Bottom panel: The Z' width as a function of g'_1 for fixed values of $M_{Z'}$.

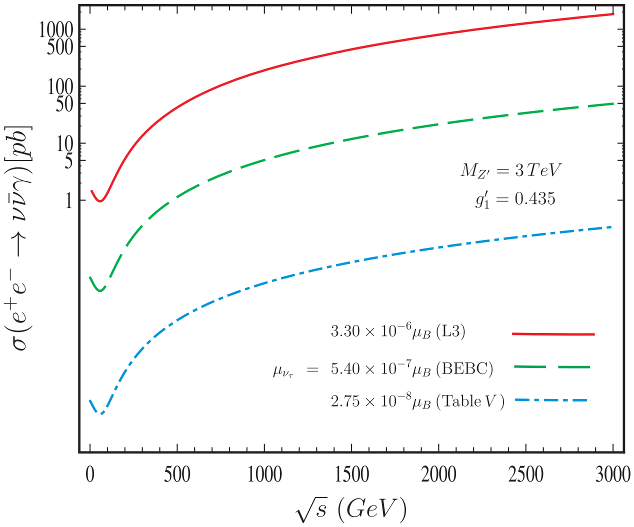

FIG. 4. The curves show the shape of $e^+e^- \to \nu_\tau\bar{\nu}_\tau\gamma$ as a function of center-of-mass energy and for different values of the μ_{ν_τ} magnetic moment.

bottom panel, we present the relative correction $\frac{\delta g_A^e}{(g_A^e)_{\rm SM}}$ as a function of g_1' and θ_{B-L}. Here it is shown that the absolute value of $\frac{\delta g_A^e}{(g_A^e)_{\rm SM}}$ increases when the parameter g_1' increases and is almost independent of the mixing angle θ_{B-L}. For $g_1' = 1$, the absolute value of $\frac{\delta g_A^e}{(g_A^e)_{\rm SM}}$ is around 4%. We find that the relative change in g_V^e is much greater than that for g_A^e for the values of the free parameters g_1' and θ_{B-L} near the end points. We conclude that the deviations of the couplings g_V^e and g_A^e from their SM values are relatively large in the parameter space (g_1', θ_{B-L}).

In Fig. 3 we present the total decay width of the Z' boson as a function of $M_{Z'}$ and the new $U(1)_{B-L}$ gauge coupling g_1', respectively, with the other parameters held fixed to three different values and $\theta_{B-L} = 10^{-3}$. From the top panel, we see that the total width of the Z' new gauge boson varies from a few to hundreds of GeV over the mass range $1000\ \mathrm{GeV} \leq M_{Z'} \leq 3500$ GeV, depending on the value of g_1', when $g_1' = 0.145$, 0.290, 0.435, respectively. In the bottom panel, a similar behavior is obtained in the range $0 \leq g_1' \leq 1$ and depends on the value of $M_{Z'} = 1000$, 2000, 3000 GeV. In both panels a clear dependence is observed on the parameters of the $U(1)_{B-L}$ model.

Figure 4 shows the total cross section for $e^+e^- \to \nu_\tau\bar{\nu}_\tau\gamma$ as a function of the center-of-mass energy $\sqrt{s}$ and different values representative of the magnetic moment that have been reported in the literature, that is, $\mu_{\nu_\tau} = 3.3 \times 10^{-6}\mu_B$(L3), $5.4 \times 10^{-7}\mu_B$[BEBC (CERN)], and $2.75 \times 10^{-8}\mu_B$ (Table V) with $M_{Z'} = 3000$ GeV and $g_1' = 0.435$. Starting from a center-of-mass energy of the order of the Z mass, a minimum around $\sqrt{s} \simeq 100$ GeV occurs due to the SM Z-boson resonance tail on the high energies. For different values of the parameter μ_{ν_τ} the shapes of the curves do not change and there is only a shift that depends on the value of the magnetic moment.

The dependence of the sensitivity limits of the magnetic moment μ_{ν_τ} with respect to the collider luminosity $\mathcal{L}$ for three different values of the center-of-mass energy, $\sqrt{s} = 1000$, 2000, 3000 GeV, heavy gauge boson mass of $M_{Z'} = 1000$, 2000, 3000 GeV, and $g_1' = 0.145$, 0.290, 0.435, respectively, is presented in Fig. 5. The figure clearly shows a strong dependence of μ_{ν_τ} with respect to $\mathcal{L}$ and the parameters of the $U(1)_{B-L}$ model. In addition, the spacings between the curves are broader for larger g_1' values, as the total width of the Z' boson increases with g_1', as shown in Fig. 3. Finally, in order to see how the total cross section $e^+e^- \to \nu_\tau\bar{\nu}_\tau\gamma$ changes with respect to the dipole moments μ_{ν_τ} and d_{ν_τ}, a three-dimensional plot is shown in Fig. 6. In this figure we consider $M_{Z'} = 3000$ GeV and $g_1' = 0.435$ in correspondence with Eq. (27).

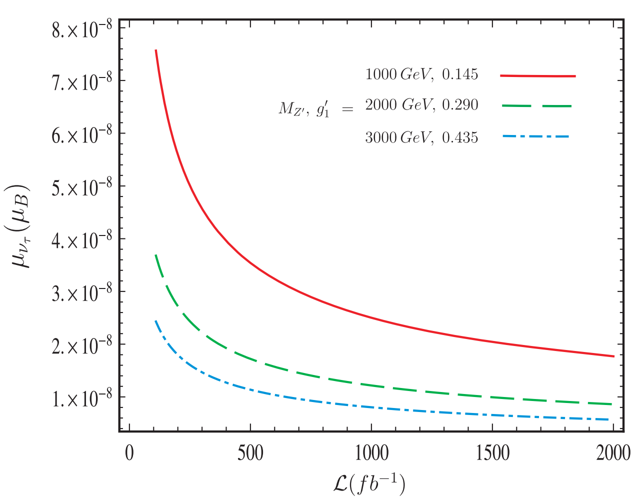

FIG. 5. Dependence of the sensitivity limits at 95% C.L. for the anomalous magnetic moment for three different values of $M_{Z'}$ and g_1' in the process $e^+e^- \to \nu_\tau\bar{\nu}_\tau\gamma$.

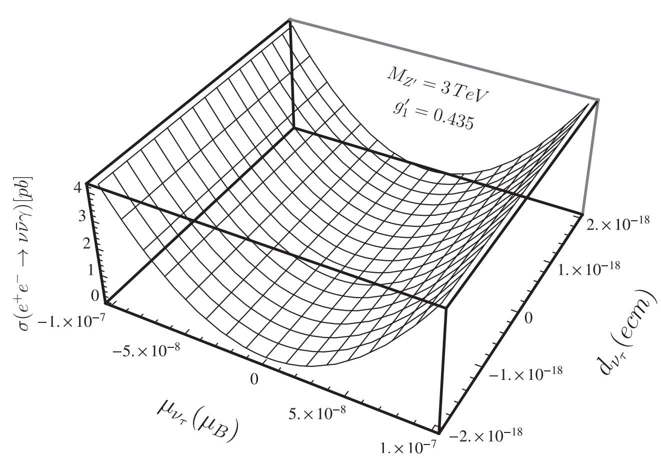

FIG. 6. The surface shows the shape for the cross section of the process $e^+e^- \to \nu_\tau\bar{\nu}_\tau\gamma$ as a function of the magnetic moment μ_{ν_τ} and the electric dipole moment d_{ν_τ}.

It is worth mentioning that by reversing the process, we can obtain specific predictions on the $U(1)_{B-L}$ models from the expression of the scattering cross section of the process $e^+e^- \to \nu_\tau\bar{\nu}_\tau\gamma$. Predictions about the models can be obtained by using the upper bound on the ν_τ magnetic moment reported in the literature by the L3 Collaboration as an input, which maximizes the total cross section, namely, $\mu_{\nu_\tau} = 3.3 \times 10^{-6}\mu_B$ (90% C.L.) [39], and using the data obtained by the ALEPH Collaboration $\sigma = (3.09 \pm 0.234)$ pb [117,118] for the cross section, where the error is statistical.

In conclusion, we have found that the process $e^+e^- \to \nu_\tau\bar{\nu}_\tau\gamma$ in the context of the Standard Model minimally extended to include massive Dirac neutrino at the high energies and luminosities expected at the ILC/CLIC colliders can be used to probe for bounds on the magnetic moment μ_{ν_τ} and electric dipole moment d_{ν_τ}. In particular, we can appreciate that the 95% C.L. sensitivity limits expected for the magnetic moment at 1000–3000 GeV center-of-mass energies already can provide proof of these bounds of order $10^{-8} - 10^{-9}$, that is to say, 2–3 orders of magnitude better than those reported in the literature; see Table II and Refs. [14–24,26–33,33,34]. Our results in Table V compare favorably with the limits obtained by the L3 Collaboration [39], and with other limits reported in the literature [14–18,26–34,38,39].

In the case of the electric dipole moment the 95% C.L. sensitivity limits at 1000–3000 GeV center-of-mass energies and integrated luminosities of 2000 fb^{-1} can provide proof of these bounds of order $10^{-19} - 10^{-20}$, that is to say, they are improved by 2–3 orders of magnitude compared to those reported in the literature; see Table III and Refs. [19–24,26,27,29,30].

The above results do not appear outside the realm of detection in future experiments with improved sensitivity. In addition, the analytical and numerical results for the cross section could be of relevance for the scientific community. Further, the results above could have possible astrophysical implications. In this regard, the stellar energy loss rate data have been used to put constraints on the properties and interaction of light particles [119–122]. In addition, one of the most interesting possibilities to use stars as particle physics laboratories [123,124] is to study the backreaction of the novel energy loss rates implied by the existence of new low-mass particles such as axions [125,126], or by non-standard neutrino properties such as the magnetic moment and electric dipole moment [10,11,127–129]. Our study complements other studies on the dipole moments of the tau neutrino at hadron and e^+e^- colliders.

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