The Theoretical Rationale of the Existence of Electric and Magnetic Fields Spreading Instantaneously

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Abstract— In this note, we show that the use of the Helmholtz vector decomposition theorem leads to the theoretical rationale of the existence of solenoidal electric and magnetic fields which can spread instantaneously.

In the comparatively recent experimental work [1] it was shown that the standard retardation condition does not take into account the complex structure of the whole electromagnetic field in the near zone. In the experiment described in [1] this unusual conclusion was obtained: the bound magnetic field in the near zone spreads with the velocity $v \gg c$, i.e., practically instantaneously. Is it possible to explain this surprising result within the framework of classical electrodynamics? In the modern literature many publications dedicated to the problem of instantaneous action at a distance (IAAD) just in classical electrodynamics do not exist. But among the few works devoted to this topic, from our point of view the most important from them are the following [2–8].

In the present work, we try to explain the result obtained in [1] taking advantage of the results of [2]. In this work the authors of [2] applied Helmholtz theorem to the vector fields in the Maxwell equations

$$\left.\begin{aligned} &\nabla\cdot\mathbf{E}=4\pi\varrho\\ &\nabla\times\mathbf{E}=-\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}\\ &\nabla\cdot\mathbf{B}=0\\ &\nabla\times\mathbf{B}=\frac{1}{c}\frac{\partial\mathbf{E}}{\partial t}+\frac{4\pi}{c}\mathbf{j}\end{aligned}\right\}, \tag{1}$$

(let us remind ourselves that Helmholtz vector decomposition theorem reads as follows[1]: *If the divergence* $D(\mathbf{r})$ *and curl* $\mathbf{C}(\mathbf{r})$ *of a vector function* $\mathbf{F}(\mathbf{r})$ *are specified, and if they both go to zero faster than* $1/r^2$ *as* $r\to\infty$, *and if* $\mathbf{F}(\mathbf{r})$ *itself tends to zero as* $r\to\infty$, *then* $\mathbf{F}(\mathbf{r})$ *is uniquely given by*

$$\mathbf{F}=-\nabla U+\nabla\times\mathbf{W}, \tag{H}$$

where

$$U(\mathbf{r})=\frac{1}{4\pi}\iiint\limits_{\text{All space}}\frac{D(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}d^3\mathbf{r}', \tag{H1}$$

and

$$\mathbf{W}(\mathbf{r})=\frac{1}{4\pi}\iiint\limits_{\text{All space}}\frac{\mathbf{C}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}d^3\mathbf{r}', \tag{H2}$$

where the vector $-\nabla U$ *is the irrotational component of* $\mathbf{F}$ *and the vector* $\nabla\times\mathbf{W}$ *is the solenoidal one*). As the result in [2] it was obtained that

$$\left.\begin{aligned} &\mathbf{E}=\mathbf{E}_i+\mathbf{E}_s\\ &\mathbf{B}=\mathbf{B}_i+\mathbf{B}_s\\ &\mathbf{j}=\mathbf{j}_i+\mathbf{j}_s\end{aligned}\right\}, \tag{2}$$

where indexes "i" and "s" signify irrotational (curl-less) and solenoidal (divergence-less) components of the vectors, respectively, and, for example,

$$\mathbf{j}_i=-\frac{1}{4\pi}\nabla\iiint\limits_{\text{All space}}\frac{\nabla'\cdot\mathbf{j}(\mathbf{r}',t)}{|\mathbf{r}-\mathbf{r}'|}d^3\mathbf{r}' \tag{3a}$$

[1] See, e.g., §1.16, p. 92 in [9]

and

$$\mathbf{j}_s = \frac{1}{4\pi}\nabla\times \iiint\limits_{\text{All space}} \frac{\nabla' \times \mathbf{j}\left(\mathbf{r}',t\right)}{|\mathbf{r}-\mathbf{r}'|} d^3\mathbf{r}'. \tag{3b}$$

It was also obtained that for the irrotational components from (1) and (2)

$$\left.\begin{array}{l} \nabla\cdot\mathbf{E}_i = 4\pi\varrho \\ \dfrac{\partial \mathbf{E}_i}{\partial t} = -4\pi\mathbf{j}_i \end{array}\right\}, \tag{4}$$

$$\left.\begin{array}{l} \nabla\cdot\mathbf{B}_i = 0 \\ \dfrac{\partial \mathbf{B}_i}{\partial t} = 0 \end{array}\right\}, \tag{5}$$

and for the solenoidal components from (1) and (2)

$$\left.\begin{array}{l} \nabla\times\mathbf{E}_s = -\dfrac{1}{c}\dfrac{\partial \mathbf{B}_s}{\partial t} \\ \nabla\times\mathbf{B}_s = \dfrac{1}{c}\dfrac{\partial \mathbf{E}_s}{\partial t} + \dfrac{4\pi}{c}\mathbf{j}_s \end{array}\right\}. \tag{6}$$

From Eq. (5) it is clear that $\mathbf{B}_i = 0$, and from Eq. (4) one can deduce that the irrotational part of the electric field is not associated with any magnetic field. But then combining Eq. (6) one can obtain wave equations for $\mathbf{E}_s$ and $\mathbf{B}_s$

$$\nabla^2\mathbf{E}_s - \frac{1}{c^2}\frac{\partial^2\mathbf{E}_s}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial \mathbf{j}_s}{\partial t}, \tag{7}$$

$$\nabla^2\mathbf{B}_s - \frac{1}{c^2}\frac{\partial^2\mathbf{B}_s}{\partial t^2} = -\frac{4\pi}{c}\nabla\times\mathbf{j}_s. \tag{8}$$

From the first equation of Eq. (4), one can deduce that $\mathbf{E}_i$ can propagate instantaneously only (i.e., with infinity velocity), whereas solutions of Eqs. (7) and (8) for $\mathbf{E}_s$ and $\mathbf{B}_s$ can characterize waves spreading with the finite velocity c. Nevertheless, wave solutions do not comprise *all* possible solutions of Eqs. (7) and (8) or solutions of the equivalent system (6).

Indeed, as it was shown in [2]

$$\mathbf{B}_s = \nabla\times\mathbf{A}_s \quad \text{and} \quad \mathbf{E}_s = -\frac{1}{c}\frac{\partial \mathbf{A}_s}{\partial t}, \tag{9}$$

where $\mathbf{A}_s$ is the solenoidal component of the magnetic vector-potential $\mathbf{A}$, and $\mathbf{A}_s$ satisfies the equation

$$\nabla^2\mathbf{A}_s - \frac{1}{c^2}\frac{\partial^2\mathbf{A}_s}{\partial t^2} = -\frac{4\pi}{c}\mathbf{j}_s. \tag{10}$$

Let us seek for *such* solution $\mathbf{A}_{s(inst)}$ of this equation which gives us

$$\frac{\partial^2\mathbf{A}_{s(inst)}}{\partial t^2} = 0. \tag{11}$$

It means that this solution of (10) is also the solution of the equation

$$\nabla^2\mathbf{A}_{s(inst)} = -\frac{4\pi}{c}\mathbf{j}_s, \tag{12}$$

and this in turn means that $\mathbf{A}_{s(inst)}$ is an instantaneous field although it is the solution of Eq. (10). In this case fields $\mathbf{E}_{s(inst)}$ and $\mathbf{B}_{s(inst)}$ obtained from Eq. (9) give us the equations

$$\nabla^2\mathbf{E}_{s(inst)} = \frac{4\pi}{c^2}\frac{\partial \mathbf{j}_s}{\partial t}, \tag{13}$$

and

$$\nabla^2\mathbf{B}_{s(inst)} = -\frac{4\pi}{c}\nabla\times\mathbf{j}_s, \tag{14}$$

that means instantaneous action for the fields $\mathbf{E}_{s(inst)}$ and $\mathbf{B}_{s(inst)}$.

This case ($\frac{\partial^2 \mathbf{A}_{s(inst)}}{\partial t^2} = 0$), considering the second equation of (9), corresponds to

$$\frac{\partial \mathbf{E}_{s(inst)}}{\partial t} = 0, \tag{15}$$

accordingly Eq. (6) convert themselves into

$$\left.\begin{aligned} \nabla \times \mathbf{E}_{s(inst)} &= -\frac{1}{c}\frac{\partial \mathbf{B}_{s(inst)}}{\partial t} \\ \nabla \times \mathbf{B}_{s(inst)} &= \frac{4\pi}{c}\mathbf{j}_s \end{aligned}\right\}. \tag{16}$$

So $\mathbf{B}_{s(inst)}$ as well as $\mathbf{j}_s$ are linear functions with respect to time. In other words, the case $\frac{\partial^2 \mathbf{A}_{s(inst)}}{\partial t^2} = 0$ means that

$$\mathbf{A}_{s(inst)}(\mathbf{r}, t) = \mathbf{A}_{s0}(\mathbf{r}) + t\mathbf{A}_{s1}(\mathbf{r}), \tag{17}$$

$$\mathbf{E}_{s(inst)} = -\frac{1}{c}\mathbf{A}_{s1}(\mathbf{r}), \tag{18}$$

$$\mathbf{B}_{s(inst)} = \nabla \times \mathbf{A}_{s0}(\mathbf{r}) + t\nabla\times\mathbf{A}_{s1}(\mathbf{r}), \tag{19}$$

$$\mathbf{j}_s = -\frac{c}{4\pi}\nabla^2\mathbf{A}_{s0}(\mathbf{r}) - \frac{c}{4\pi}t\nabla^2\mathbf{A}_{s1}(\mathbf{r}). \tag{20}$$

So one can infer that instantaneous field $\mathbf{A}_{s(inst)}$ generates instantaneous fields $\mathbf{E}_{s(inst)}$ and $\mathbf{B}_{s(inst)}$, and the current $\mathbf{j}_s$ generating the field $\mathbf{A}_{s(inst)}$ must be linear functions with respect to time.

It is obvious that our field $\mathbf{B}_{s(inst)}$ can play the role of the *bound magnetic field in the near zone spreading with the infinite velocity (i.e., instantaneously)* that has been observed in [1]. The authors of the paper [1] obtained experimentally the propagation speed of bound magnetic field. The measurements consisted of measuring the dependence of the propagation time between emitting multisection loop antenna and receiving multisection loop antenna, on the distance between them. The authors conclude that experimental data do not support the validity of the standard retardation constraint $v = c$, generally accepted in respect to bound fields. In contrast, in the paper [1] one can see nearly perfect coincidence between experimental data and theoretical prediction when the retardation parameter v for bound fields highly exceeds the velocity of light, i.e., $v > 10c$. Thus it can be considered that the existence of superluminal (de facto instantaneous) bound magnetic field generated by the current linearly dependent on time, recently experimentally demonstrated in [1], now is theoretically substantiated.

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