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NECESSITY OF SIMULTANEOUS CO-EXISTENCE OF INSTANTANEOUS AND RETARDED INTERACTIONS IN CLASSICAL ELECTRODYNAMICS

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We consider the electromagnetic field of a charge moving with a constant acceleration along an axis. We find that this field obtained from the Liénard–Wiechert potentials does not satisfy Maxwell equations if one considers exclusively a retarded interaction. We show that if and only if one takes into account both retarded interaction and direct interaction (so-called "instantaneous action at a distance") the field produced by an accelerated charge satisfies Maxwell equations.

1. Introduction

The problem of a calculation of the potentials and the fields created by a point charge moving with an acceleration was first raised approximately 100 years ago by Liénard and Wiechert[1] and is still pertinent today. The question concerning the choice of a correct way of obtaining these fields seemed to have been solved finally (see, e.g. Landau and Lifshitz's well-known book[2]). However, many authors (see, e.g. Refs. 3–6 and references therein) have recently taken up this problem once more, a problem which had been abandoned by contemporary physics some time ago. In this paper we shall establish the assertion made in the abstract.

It is well-known that the electromagnetic field created by an arbitrarily moving charge

$$\mathbf{E}(\mathbf{r},t) = q\left\{\frac{\left(\mathbf{R}-R\frac{\mathbf{V}}{c}\right)\left(1-\frac{V^2}{c^2}\right)}{\left(R-\mathbf{R}\frac{\mathbf{V}}{c}\right)^3}\right\}_{t_0} + q\left\{\frac{\left[\mathbf{R}\times\left[\left(\mathbf{R}-R\frac{\mathbf{V}}{c}\right)\times\frac{\dot{\mathbf{V}}}{c^2}\right]\right]}{\left(R-\mathbf{R}\frac{\mathbf{V}}{c}\right)^3}\right\}_{t_0}, \tag{1}$$

$$\mathbf{B}(\mathbf{r},t) = \left\{\left[\frac{\mathbf{R}}{R}\times\mathbf{E}\right]\right\}_{t_0}, \tag{2}$$

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was obtained directly from the Liénard–Wiechert potentials.[2]

$$\varphi(\mathbf{r},t)=\left\{\frac{q}{(R-\mathbf{R}\frac{\mathbf{V}}{c})}\right\}_{t_0}, \qquad \mathbf{A}(\mathbf{r},t)=\left\{\frac{q\mathbf{V}}{c(R-\mathbf{R}\frac{\mathbf{V}}{c})}\right\}_{t_0}. \tag{3}$$

The notation $\{\cdots\}_{t_0}$ means that all functions of x,y,z,t in the parenthesis $\{\ \}$ are taken at the moment of time $t_0(x,y,z,t)^2$ (the instant t_0 is determined from condition (8), see below).

Usually, the first terms of the right-hand sides (r.h.s.) of (1) and (2) are called "velocity fields" and the second ones are called "acceleration fields."

It was recently claimed by E. Comay[7] that "*... acceleration fields by themselves do not satisfy Maxwell's equations.*[8] *Only the sum of velocity fields and acceleration fields satisfies Maxwell's equations.*" We wish to argue that this sum *does not satisfy* Maxwell's equations

$$\nabla\cdot\mathbf{E}=4\pi\varrho\,, \tag{4}$$

$$\nabla\cdot\mathbf{B}=0\,, \tag{5}$$

$$\nabla\times\mathbf{B}=\frac{4\pi}{c}\mathbf{j}+\frac{1}{c}\frac{\partial\mathbf{E}}{\partial t}\,, \tag{6}$$

$$\nabla\times\mathbf{E}=-\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}\,, \tag{7}$$

in the case when one takes into consideration *exclusively* a retarded interaction.

The remainder of our paper is organized as follows: In Sec. 2 we derive the fields **E** and **B** taking into account exclusively the *implicit* dependence of the potentials φ and **A** on time t. In Sec. 3 we prove that the field obtained from the Liénard–Wiechert potentials does not satisfy Maxwell equations if one considers exclusively a retarded interaction (in other words, if one considers only the *implicit* dependence of the potentials on observation time t). In Sec. 4 we consider another way of obtaining the fields **E** and **B**. This way is based on a different type of calculation of the derivatives $\partial\{\ \}/\partial t$ and $\partial\{\ \}/\partial x_i$ in which the functions φ and **A** are considered as functions with a *double* dependence on (t,x,y,z): implicit and explicit *simultaneously*. By this way, one obtains formally the *same* expressions (1) and (2) for the fields. If one uses *this* manner to verify the validity of Maxwell's equations, one finds that fields (1) and (2) satisfy these equations. In this section, we shall show that this way does not correspond to a *pure* retarded interaction between the charge and the point of observation. Section 5 closes the paper.

2. Derivation of the Fields E and B Taking into Account the Retarded Interaction Only

Let us try to derive the formulas (1), (2) for the electric (**E**) and magnetic (**B**) fields taking into account that the *state* of the fields **E** and **B** at the instant t must be *completely* determined by the *state* of the charge at the instant t_0. The instant t_0 is determined from the condition [see Eq. (63.1) of Ref. 2]:

$$t_0 = t - \tau = t - \frac{R(t_0)}{c} \,. \tag{8}$$

Here $\tau = R(t_0)/c$ is the so called "retarded time," $R = |\mathbf{R}|$, $\mathbf{R}$ is the vector connecting the site $\mathbf{r}_0(x_0, y_0, z_0)$ of the charge q at the instant t_0 with the point of observation $\mathbf{r}(x, y, z)$.

All the quantities on the *rhs* of (3) must be evaluated at the time t_0 [see the text after Eq. (63.5) in Ref. 2], which, in turn, depends on x, y, z, t:

$$t_0 = f(x, y, z, t) \,. \tag{9}$$

Let us, to be more specific, turn to Landau and Lifshitz who write (Ref. 2, p. 161):[a] "*To calculate the intensities of the electric and magnetic fields from the formulas*

$$\mathbf{E} = -\nabla\varphi - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} \,, \qquad \mathbf{B} = [\nabla \times \mathbf{A}] \,, \tag{10}$$

we must differentiate φ *and* $\mathbf{A}$ *with respect to the coordinates* x, y, z *of the point, and the time* t *of observation. But the formulas* (3) *express the potentials as a functions of* t_0, and only through *the relation* (8) as implicit *functions of* x, y, z, t. *Therefore to calculate the required derivatives we must first calculate the derivatives of* t_0."

Now, following this note of Landau and Lifshitz, we can construct a scheme of calculating the required derivatives, taking into account that φ and $\mathbf{A}$ *must not* depend on x, y, z, t *explicitly*:

$$\left.\begin{aligned} \frac{\partial \varphi}{\partial x_i} &= \frac{\partial \varphi}{\partial t_0}\frac{\partial t_0}{\partial x_i} \\ \frac{\partial \mathbf{A}}{\partial t} &= \frac{\partial \mathbf{A}}{\partial t_0}\frac{\partial t_0}{\partial t} \\ \frac{\partial A_k}{\partial x_i} &= \frac{\partial A_k}{\partial t_0}\frac{\partial t_0}{\partial x_i} \end{aligned}\right\} \,. \tag{11}$$

To obtain Eqs. (1) and (2), let us rewrite Eqs. (10) taking into account Eqs. (11):[b]

$$\mathbf{E} = -\nabla\varphi - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} = -\frac{\partial \varphi}{\partial t_0}\nabla t_0 - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t_0}\frac{\partial t_0}{\partial t} \,, \tag{12}$$

$$\mathbf{B} = [\nabla \times \mathbf{A}] = \left[\nabla t_0 \times \frac{\partial \mathbf{A}}{\partial t_0}\right] . \tag{13}$$

[a] We use here our numeration of formulas: our (3) is (63.5) of Ref. 2, (8) is (63.1) of Ref. 2.
[b] In Eqs. (12) and (13) we have used the well-known formulas of the vectorial analysis:

$$\nabla u = \frac{\partial u}{\partial \xi}\nabla\xi \,, \quad \text{and} \quad [\nabla \times \mathbf{f}] = \left[\nabla\xi \times \frac{\partial \mathbf{f}}{\partial \xi}\right] ,$$

where $u = u(\xi)$, $\mathbf{f} = \mathbf{f}(\xi)$ and $\xi = \xi(x, y, z)$.

To calculate Eqs. (12) and (13) we use relations $\partial t_0/\partial t$ and $\partial t_0/\partial x_i$ obtained in Ref. 2:

$$\frac{\partial t_0}{\partial t} = \frac{R}{R - \mathbf{R}\mathbf{V}/c} \quad \text{and} \quad \frac{\partial t_0}{\partial x_i} = -\frac{x_i - x_{0i}}{c[R - \mathbf{R}\mathbf{V}/c]} . \tag{14}$$

From Eqs. (3) we find:

$$\frac{\partial \varphi}{\partial t_0} = -\frac{q}{(R - \mathbf{R}\boldsymbol{\beta})^2}\left(\frac{\partial R}{\partial t_0} - \frac{\partial \mathbf{R}}{\partial t_0}\boldsymbol{\beta} - \mathbf{R}\frac{\partial \boldsymbol{\beta}}{\partial t_0}\right) , \tag{15}$$

where $\boldsymbol{\beta} = \mathbf{V}/c$. Hence, taking into account that[c]

$$\frac{\partial R}{\partial t_0} = -c\,, \quad \frac{\partial \mathbf{R}}{\partial t_0} = -\frac{\partial \mathbf{r}_0}{\partial t_0} = -\mathbf{V}(t_0) \quad \text{and} \quad \frac{\partial \mathbf{V}}{\partial t_0} = \dot{\mathbf{V}}\,,$$

we have (after an algebraic simplification):

$$\frac{\partial \varphi}{\partial t_0} = \frac{qc(1 - \beta^2 + \mathbf{R}\dot{\boldsymbol{\beta}}/c)}{(R - \mathbf{R}\boldsymbol{\beta})^2} . \tag{16}$$

In turn

$$\frac{\partial \mathbf{A}}{\partial t_0} = \frac{\partial \varphi}{\partial t_0}\boldsymbol{\beta} + \varphi\dot{\boldsymbol{\beta}}. \tag{17}$$

Putting φ from Eqs. (3), (16) and (17) together, we have (after simplification):

$$\frac{\partial \mathbf{A}}{\partial t_0} = qc\,\frac{\boldsymbol{\beta}(1 - \beta^2 + \mathbf{R}\dot{\boldsymbol{\beta}}/c) + (\dot{\boldsymbol{\beta}}/c)(R - \mathbf{R}\boldsymbol{\beta})}{(R - \mathbf{R}\boldsymbol{\beta})^2} . \tag{18}$$

Finally, substituting Eqs. (14), (16) and (18) in Eq. (12) we obtain

$$\begin{aligned}\mathbf{E} &= \frac{qc(1 - \beta^2 + \mathbf{R}\dot{\boldsymbol{\beta}}/c)}{(R - \mathbf{R}\boldsymbol{\beta})^2}\left(-\frac{\mathbf{R}}{c(R - \mathbf{R}\boldsymbol{\beta})}\right) \\ &\quad - q\,\frac{\boldsymbol{\beta}(1 - \beta^2 + \mathbf{R}\dot{\boldsymbol{\beta}}/c) + (\dot{\boldsymbol{\beta}}/c)(R - \mathbf{R}\boldsymbol{\beta})}{(R - \mathbf{R}\boldsymbol{\beta})^2}\left(\frac{R}{R - \mathbf{R}\boldsymbol{\beta}}\right) \\ &= q\,\frac{\mathbf{R}(1 - \beta^2 + \mathbf{R}\dot{\boldsymbol{\beta}}/c) - R\boldsymbol{\beta}(1 - \beta^2 + \mathbf{R}\dot{\boldsymbol{\beta}}/c) - (R\dot{\boldsymbol{\beta}}/c)(R - \mathbf{R}\boldsymbol{\beta})}{(R - \mathbf{R}\boldsymbol{\beta})^3} .\end{aligned} \tag{19}$$

Grouping together all terms with acceleration, one can reduce this expression to

$$\mathbf{E} = q\,\frac{\left(\mathbf{R} - R\frac{\mathbf{V}}{c}\right)\left(1 - \frac{V^2}{c^2}\right)}{\left(R - \mathbf{R}\frac{\mathbf{V}}{c}\right)^3} + q\,\frac{(\mathbf{R}\dot{\boldsymbol{\beta}}/c)(\mathbf{R} - R\boldsymbol{\beta}) - (R\dot{\boldsymbol{\beta}}/c)(R - \mathbf{R}\boldsymbol{\beta})}{(R - \mathbf{R}\boldsymbol{\beta})^3} . \tag{20}$$

Now, using the formula of the double vectorial product, it is not worth reducing the numerator of the second term of Eq. (20) to $[\mathbf{R} \times [(\mathbf{R} - R\boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}/c]]$. As a result we have Eq. (1).

[c]This follows from the expressions $R = c(t - t_0)$ and $\mathbf{R} = \mathbf{r} - \mathbf{r}_0(t_0)$. See Ref. 2.

Analogically, substituting Eqs. (14) and (18) in Eq. (13) we obtain

$$\mathbf{B} = \left[\frac{\mathbf{R}}{R} \times q \, \frac{-R\boldsymbol{\beta}(1-\beta^2+\mathbf{R}\dot{\boldsymbol{\beta}}/c) - (R\dot{\boldsymbol{\beta}}/c)(R-\mathbf{R}\boldsymbol{\beta})}{(R-\mathbf{R}\boldsymbol{\beta})^3} \right] . \qquad (21)$$

If we add $\mathbf{R}(1-\beta^2+\mathbf{R}\dot{\boldsymbol{\beta}}/c)$ to the numerator of the second term of the vectorial product (21)[d] we obtain Eq. (2) [see Eq. (19)]

In the next section we shall consider a charge moving with a constant acceleration along the X axis and we shall show that the Eq. (7) *is not satisfied* if one substitutes $\mathbf{E}$ and $\mathbf{B}$ from Eqs. (1) and (2) in Eq. (7) and *takes into consideration exclusively a retarded interaction.* To verify this we have to find the derivatives of x-, y-, z-components of the fields $\mathbf{E}$ and $\mathbf{B}$ with respect to the time t and the coordinates x, y, z. The functions $\mathbf{E}$ and $\mathbf{B}$ depend on x,y, z, t through t_0 from the conditions (8)–(9). In other words, we shall show that *these fields* $\mathbf{E}$ *and* $\mathbf{B}$ *do not satisfy the Maxwell equations if the differentiation rules (11) that were applied to* φ *and* $\mathbf{A}$ *(to obtain* $\mathbf{E}$ *and* $\mathbf{B}$*) are applied identically to* $\mathbf{E}$ *and* $\mathbf{B}$.

3. Does the Retarded Electromagnetic Field of a Charge Moving with a Constant Acceleration Satisfy Maxwell Equations?

Let us consider a charge q moving with a constant acceleration along the X axis. In this case its velocity and acceleration have only x-components, respectively $\mathbf{V}(V,0,0)$ and $\mathbf{a}(a,0,0)$. Now we rewrite the Eqs. (1) and (2) by components:

$$E_x(x,y,z,t) = q\left\{\frac{(V^2-c^2)[RV-c(x-x_0)]}{[(cR-V(x-x_0)]^3}\right\}_{t_0} + q\left\{\frac{ac[(x-x_0)^2-R^2]}{[(cR-V(x-x_0)]^3}\right\}_{t_0} , \qquad (22)$$

$$E_y(x,y,z,t) = -q\left\{\frac{c(V^2-c^2)(y-y_0)}{[(cR-V(x-x_0)]^3}\right\}_{t_0} + q\left\{\frac{ac(x-x_0)(y-y_0)}{[(cR-V(x-x_0)]^3}\right\}_{t_0} , \qquad (23)$$

$$E_z(x,y,z,t) = -q\left\{\frac{c(V^2-c^2)(z-z_0)}{[(cR-V(x-x_0)]^3}\right\}_{t_0} + q\left\{\frac{ac(x-x_0)(z-z_0)}{[(cR-V(x-x_0)]^3}\right\}_{t_0} , \qquad (24)$$

$$B_x(x,y,z,t) = 0 , \qquad (25)$$

[d]The meaning of Eq. (21) does not change because $[\mathbf{R}\times\mathbf{R}]=0$.

$$B_y(x,y,z,t) = q\left\{\frac{V(V^2-c^2)(z-z_0)}{[(cR-V(x-x_0)]^3}\right\}_{t_0} - q\left\{\frac{acR(z-z_0)}{[(cR-V(x-x_0)]^3}\right\}_{t_0}, \tag{26}$$

$$B_z(x,y,z,t) = -q\left\{\frac{V(V^2-c^2)(y-y_0)}{[(cR-V(x-x_0)]^3}\right\}_{t_0} + q\left\{\frac{acR(y-y_0)}{[(cR-V(x-x_0)]^3}\right\}_{t_0}. \tag{27}$$

Obviously, these components are functions of x, y, z, t through t_0 from the conditions (8) and(9). This means that when we substitute the field components given by Eqs. (22)–(27) in the Maxwell Eqs. (4) and (7), we once again have to use the differentiation rules as in (11):

$$\left.\begin{aligned}\frac{\partial E\{\text{or } B\}_k}{\partial t} &= \frac{\partial E\{\text{or } B\}_k}{\partial t_0}\frac{\partial t_0}{\partial t}\\ \frac{\partial E\{\text{or } B\}_k}{\partial x_i} &= \frac{\partial E\{\text{or } B\}_k}{\partial t_0}\frac{\partial t_0}{\partial x_i}\end{aligned}\right\}, \tag{28}$$

where k and x_i are x,y, z.

Remember that we are considering the case with $\mathbf{V} = (V, 0, 0)$, so, one obtains

$$\frac{\partial t_0}{\partial t} = \frac{R}{R-(x-x_0)V/c}, \quad \text{and} \quad \frac{\partial t_0}{\partial x_i} = -\frac{x_i - x_{0i}}{c[R-(x-x_0)V/c]}. \tag{29}$$

Let us rewrite Eq. (7) by components taking into account the rules (28) and Eq. (25):

$$\frac{\partial E_z}{\partial t_0}\frac{\partial t_0}{\partial y} - \frac{\partial E_y}{\partial t_0}\frac{\partial t_0}{\partial z} = 0, \tag{30}$$

$$\frac{\partial E_x}{\partial t_0}\frac{\partial t_0}{\partial z} - \frac{\partial E_z}{\partial t_0}\frac{\partial t_0}{\partial x} + \frac{1}{c}\frac{\partial B_y}{\partial t_0}\frac{\partial t_0}{\partial t} = 0, \tag{31}$$

$$\frac{\partial E_y}{\partial t_0}\frac{\partial t_0}{\partial x} - \frac{\partial E_x}{\partial t_0}\frac{\partial t_0}{\partial y} + \frac{1}{c}\frac{\partial B_z}{\partial t_0}\frac{\partial t_0}{\partial t} = 0. \tag{32}$$

In order to calculate the derivatives $\partial E(\text{or } B)_k/\partial t_0$ we need the values of the expressions $\partial V/\partial t_0$, $\partial x_0/\partial t_0$ and $\partial R/\partial t_0$. In our case we have to use[e]

$$\frac{\partial R}{\partial t_0} = -c, \qquad \frac{\partial x_0}{\partial t_0} = V, \quad \text{and} \quad \frac{\partial V}{\partial t_0} = a. \tag{33}$$

[e]See footnote c.

Now, using Eqs. (29) and (33), we want to verify the validity of Eqs. (30)–(32). The result of the verification is as follows:

$$\frac{\partial E_z}{\partial t_0}\frac{\partial t_0}{\partial y} - \frac{\partial E_y}{\partial t_0}\frac{\partial t_0}{\partial z} = 0\,, \tag{34}$$

$$\frac{\partial E_x}{\partial t_0}\frac{\partial t_0}{\partial z} - \frac{\partial E_z}{\partial t_0}\frac{\partial t_0}{\partial x} + \frac{1}{c}\frac{\partial B_y}{\partial t_0}\frac{\partial t_0}{\partial t} = -\frac{ac(z-z_0)}{[cR - V(x-x_0)]^3}\,, \tag{35}$$

$$\frac{\partial E_y}{\partial t_0}\frac{\partial t_0}{\partial x} - \frac{\partial E_x}{\partial t_0}\frac{\partial t_0}{\partial y} + \frac{1}{c}\frac{\partial B_z}{\partial t_0}\frac{\partial t_0}{\partial t} = \frac{ac(y-y_0)}{[cR - V(x-x_0)]^3}\,. \tag{36}$$

The verification[f] shows that Eq. (30) is valid. But instead of Eqs. (31) and (32) we have Eqs. (35) and (36) respectively. A reader has to agree that this result is rather unexpected.

However, another way to obtain the fields (1) and (2) exists. If one uses *this* manner to verify the validity of Maxwell's equations, one finds that fields (1) and (2) satisfy these equations. In the next section we shall consider this way in detail and we shall show that it does not correspond to a *pure* retarded interaction between the charge and the point of observation.

4. Double (Implicit and Explicit) Dependence of φ, A, E and B on t and x_i. Total Derivatives: Mathematical and Physical Aspects

First at all, let us consider in detail Landau's method[2] of obtaining the derivatives $\partial t_0/\partial t$ and $\partial t_0/\partial x_i$. Landau and Lifshitz considered two different expressions of the function R:

$$R = c(t - t_0)\,, \quad \text{where} \quad t_0 = f(x, y, z, t)\,, \tag{37}$$

and

$$R = [(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2]^{1/2}\,, \quad \text{where} \quad x_{0i} = f_i(t_0)\,. \tag{38}$$

Then one calculates the derivatives ($\partial/\partial t$ and $\partial/\partial x_i$) of functions (37) and (38), and equating the results, obtains $\partial t_0/\partial t$ and $\partial t_0/\partial x_i$. While Landau and Lifshitz use here a symbol ∂ (see the expression before Eq. (63.6) in Ref. 2) in order to emphasize that R depends also on other independent variables x, y, z, it is easy to show that they calculate here *total* derivatives of the functions (37), (38) *with respect to* t and x_i. The point is that if a *given* function is expressed by two different types of functional dependencies, then exclusively *total* derivatives of these expressions

[f]There is another manner of verifying the validity of Eqs. (30)–(32). If one substitutes **E** and **B** from (10) in Eq. (7), one only has to satisfy oneself that the operators "$\nabla\times$" and "$\partial/\partial t$" commute. In our case, because of $\mathbf{V} = (V, 0, 0)$ and $\mathbf{A} = (A_x, 0, 0)$, it means the commutation of the operators $\partial/\partial y$(or z) and $\partial/\partial t$. The verification shows that these operators do not commute if one uses the rules (11).

with respect to a *given* variable can be equated (contrary to the *partial* ones). Here we adduce the scheme[g] which was used in Ref. 2 to obtain $\partial t_0/\partial t$ and $\partial t_0/\partial x_i$:

$$\left[\begin{array}{ccccc} \underbrace{\frac{\partial R}{\partial t}_{(=c)} + \frac{\partial R}{\partial t_0}_{(=-c)}\frac{\partial t_0}{\partial t}} & = & \underbrace{\frac{dR}{dt}} & = & \underbrace{\sum_k \frac{\partial R}{\partial x_{0k}}\frac{\partial x_{0k}}{\partial t_0}\frac{\partial t_0}{\partial t}} \\ \uparrow & & \uparrow & & \uparrow \\ R\{t, t_0(x_i,t)\} & & = R(t_0) = & & R\{x_i, x_{0i}[t_0(x_i,t)]\} \\ \Updownarrow & & \Updownarrow & & \Updownarrow \\ c(t-t_0) & & = R(t_0) = & & \left\{\sum_i [(x_i - x_{0i}(t_0)]^2\right\}^{1/2} \\ \downarrow & & \downarrow & & \downarrow \\ \overbrace{\frac{\partial R}{\partial t_0}_{(=-c)}\frac{\partial t_0}{\partial x_i}} & = & \overbrace{\frac{dR}{dx_i}} & = & \overbrace{\frac{\partial R}{\partial x_i}_{\left(=\frac{x_i-x_{0i}}{R}\right)} + \sum_k \frac{\partial R}{\partial x_{0k}}\frac{\partial x_{0k}}{\partial t_0}\frac{\partial t_0}{\partial x_i}} \end{array}\right]. \tag{39}$$

If one takes into account that $\partial t/\partial x_i = \partial x_i/\partial t = 0$, as a result obtains the same values of the derivatives which have been obtained in (14).

Let us now, as it was mentioned above in the end of Sec. 3, calculate the expressions (10) taking into consideration that the functions φ and $\mathbf{A}$ depend on t (or on x_i)[h] implicitly and explicitly *simultaneously*. In this case we have:

$$\frac{\partial \varphi}{\partial x_i} = -\frac{q}{(R - \mathbf{R}\boldsymbol{\beta})^2}\left(\frac{\partial R}{\partial x_i} - \frac{\partial \mathbf{R}}{\partial x_i}\boldsymbol{\beta} - \mathbf{R}\frac{\partial \boldsymbol{\beta}}{\partial x_i}\right), \tag{40}$$

$$\frac{\partial \varphi}{\partial t} = -\frac{q}{(R - \mathbf{R}\boldsymbol{\beta})^2}\left(\frac{\partial R}{\partial t} - \frac{\partial \mathbf{R}}{\partial t}\boldsymbol{\beta} - \mathbf{R}\frac{\partial \boldsymbol{\beta}}{\partial t}\right), \tag{41}$$

and

$$\frac{\partial \mathbf{A}}{\partial t} = \frac{\partial \varphi}{\partial t}\boldsymbol{\beta} + \varphi\frac{\partial \boldsymbol{\beta}}{\partial t}, \tag{42}$$

where

$$\frac{\partial \boldsymbol{\beta}}{\partial t} = \frac{\partial \boldsymbol{\beta}}{\partial t_0}\frac{\partial t_0}{\partial t} \quad \text{and} \quad \frac{\partial \boldsymbol{\beta}}{\partial x_i} = \frac{\partial \boldsymbol{\beta}}{\partial t_0}\frac{\partial t_0}{\partial x_i}. \tag{43}$$

[g]In this scheme we have used a symbol d for a total derivative. In the original text[2] we have

$$\frac{\partial R}{\partial t} = \frac{\partial R}{\partial t_0}\frac{\partial t_0}{\partial t} = -\frac{\mathbf{RV}}{R}\frac{\partial t_0}{\partial t} = c\left(1 - \frac{\partial t_0}{\partial t}\right),$$

$$\nabla t_0 = -\frac{1}{c}\nabla R(t_0) = -\frac{1}{c}\left(\frac{\partial R}{\partial t_0}\nabla t_0 + \frac{\mathbf{R}}{R}\right).$$

[h]This depends on the choice of the expression for R in (37) and (38).

Now, let us consider *all* derivatives in (10), (40)–(43) as *total* derivatives with respect to t and x_i. Then, if we substitute the expressions (40)–(43) in (10) (of course, taking into account either *l.h.s.* or *r.h.s.* of the scheme (39)), we obtain formally the *same* expressions for the fields (1) and (2)! Then if one substitutes the fields (1) and (2) in Maxwell's equation (7), considering *all* derivatives in (7) as *total* ones and, of course, considering the functions **E** and **B** as functions with both implicit and explicit dependence on t (or on x_i), one can see that Eq. (7) is satisfied!

5. Conclusion

If we consider *only* the implicit functional dependence of **E** and **B** with respect to the time t this means that we describe *exclusively* the retarded interaction: the electromagnetic perturbation created by the charge at the instant t_0 *reaches* the point of observation (x, y, z) after the time $\tau = R(t_0)/c$. Surprisingly, the Maxwell equations *are not* satisfied in this case!

If we take into account a possible *explicit* functional dependence of **E** and **B** with respect to the time t, *together* with the *implicit* dependence, the Maxwell equations are satisfied. The explicit dependence of **E** and **B** on t means that, contrary to the implicit dependence, *there is not* a retarded time for electromagnetic perturbation *to reach* the point of observation. A possible interpretation may be an action-at-a-distance phenomenon, as a full-value solution of the Maxwell equations within the framework of the so called "dualism concept."[9,10,i] This interpretation differs from the well-known "retarded action at a distance" concept (see, e.g. Refs. 12 and 13 and references therein) and could be an alternative point of view. In other words, there is a *simultaneous* and *independent* coexistence of **instantaneous** and **retarded interactions** which cannot be reduced to each other.

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[i] In fact, a considerable number of works have recently been published which directly declare: classical electrodynamics must be *reconsidered*. See, e.g. Ref. 11 and corresponding references.

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