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REPLY TO COMMENT

Reply to Comment on 'Electromagnetic potentials without gauge transformation'

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Abstract
This is a reply to the criticism from Engelhardt and Onoochin of our work (2011 *Phys. Scr.* **84** 015009): a general argument for the possibility of different solutions in different gauges unrelated to gauge transformation; a result that has been given by Engelhardt and Onoochin using examples. For this reason we are not in any sense trying to refute the statements made by Engelhardt and Onoochin, instead we are offering a possible theoretical explanation of their results.

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To solve Maxwell's equations, the introduction of scalar and vector potentials φ, A is natural. We call these potentials 'g-potentials'. When these potentials are introduced it is clear that there is freedom in its definition. That is, we can always introduce new g-potentials φ_0, $\mathbf{A}_0$ without changing the electric and magnetic fields using the following transformation, known as the 'gauge transformation':

$$\varphi = \varphi_0 - \frac{\partial G}{\partial t}, \quad \mathbf{A} = \mathbf{A}_0 + \nabla G. \tag{1}$$

Hence we have a transformation theory for the g-potentials. Along with these potentials, in order to simplify the field equations that define them, a new condition that involves the first-order partial derivatives of the potentials—known as the 'gauge condition'—is required. To some extent the gauge condition is arbitrary, but its choice leads to different sets of field equations for the potentials. But this is of no harm because the g-potentials can be defined in such a way that the gauge condition is satisfied using, if necessary, a gauge transformation. Hence we first introduce freedom in the definition of the potentials as a gauge transformation and then we realize that we can satisfy any gauge condition using a gauge transformation defined by its function G. Obviously, because of the freedom in their definition the g-potentials are considered as artifacts of the mathematical procedure without any physical relevance in the classical domain.

The foregoing is a summary of the conventional wisdom about the widespread method to solve Maxwell's equations by means of the g-potentials, and can be read in any textbook on electromagnetic theory (see, e.g., [1, pp 239–241]). In the critical comment by Engelhardt and Onoochin [2] on the paper [3] it is proved, in the right manner, that the field equations deduced for the introduced gauge invariant potentials φ_g, A_g are formally the same as those for the g-potentials in the Coulomb gauge. However, we believe that this is not a criticism because the authors overlooked the main achievement of [3] that for us relies on two independent points:

(i) A general argument for the possibility of different solutions in different gauges unrelated by a gauge transformation. A result that has been given by Engelhardt and Onoochin using examples. For this reason we cannot believe that in any sense we are trying to refute statements in [2], instead we are offering a possible theoretical explanation of their results.
(ii) The proof that the new potentials are gauge invariant quantities.

These points are logically independent and become in their interrelation a general criticism of the generally accepted wisdom. But while (i) is a criticism of a negative character against the possibility of gauge transformations in general, point (ii) is positive, because it offers a new way to look at the potentials. It is precisely because of (ii) that we cannot believe that, indeed, what Engelhardt and Onoochin say in [2] is really a criticism. As a matter of fact it is very clear that the g-potentials in the Coulomb gauge are *not gauge invariant* while the potentials we have introduced are. Therefore it is clear that what Engelhardt and Onoochin are saying is:

(a) The usual g-potentials in the Coulomb gauge are gauge invariant.
(b) The new potentials are not gauge invariant, and the proof of it is flawed.

Obviously, we can see that statement (a) cannot be correct and after a closer look at the proof in [3] of gauge invariance we cannot find any flaw. Hence we conclude that what Engelhardt and Onoochin had done is a proof of the formal resemblance of the g-potentials in the Coulomb gauge with our gauge invariant potentials. That is, it seems that in the Coulomb gauge the g-potentials satisfy the same field equations as the gauge invariant potentials, but contrary to the g-potentials in the Coulomb gauge, the gauge invariant potentials cannot be related to any other potentials in any other gauge. This is a neat coincidence indeed, because it is known that in the Coulomb gauge, *a fortiori* with the new gauge invariance potentials, when the electromagnetic field is quantized, only physical modes are allowed to propagate (see, e.g., [4, p 107]). There are more reasons to believe that this coincidence with the Coulomb gauge cannot be accidental. It has been remarked in the corresponding literature that it is precisely in the Coulomb gauge that the theoretical predictions for electromagnetically bound systems are correct at the quantum level, while those predictions done in the Lorenz gauge are incorrect; see, for example, [5]. Obviously, these are only remarks and more research in that direction is needed. However, it is necessary to stress the idea of gauge invariance of the new potentials, showing that in this sense they are more physical than the usual ones.

There is another criticism in the comment by Engelhardt and Onoochin that is in need of an answer. They claim that a gauge has been used in an implicit manner. This is incorrect. We have explained at the beginning what the conventional theory of gauge conditions and gauge transformations is, and in our paper [3] we have not used it. Instead we have used an independent result: the Helmholtz theorem which is a mathematical result unrelated to the conventional gauge theory of classical electrodynamics. Indeed the condition $\nabla \cdot \mathbf{A}_s = 0$ appears in the conditions of the theorem, and Engelhardt and Onoochin see the Coulomb gauge here. But what about the condition $\nabla \times \mathbf{A}_i = 0$? Is this a gauge condition, too? Obviously, it could be some sort of gauge condition, but in any case it is new, and what is more important is that the potentials introduced using these 'gauge conditions' are gauge invariant, and so the proof of gauge invariance is where the most striking difference with conventional wisdom lies.

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