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The inertial property of approximately inertial frames of reference

Andrew E Chubykalo[1], **Augusto Espinoza**[1] and **B P Kosyakov**[2]

[1] Escuela de Física, Universidad Autónoma de Zacatecas, Apartado Postal C-580, Zacatecas 98068, Zacatecas, Mexico

[2] Russian Federal Nuclear Center, Sarov, 607190 Nizhnii Novgorod Region, Russia

E-mail: achubykalo@yahoo.com.mx, drespinozag@yahoo.com.mx and kosyakov@vniief.ru

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Abstract

Is it possible to compare approximately inertial frames in the inertial property? If this is the case, the inertial property becomes a measurable quantity. We give a positive answer to this question, and discuss the general principle of design of devices for making the required measurements. This paper is intended for advanced undergraduate and graduate students in high energy physics and relativity. Our aim is twofold: (i) to provide a deeper insight into the essentials of classical dynamics, and (ii) to give impetus to ingenious young people to devise new clever, useful and highly sensitive tools for measuring the inertial property following the pattern outlined in the present discussion.

1. Introduction

In laboratory practice, the most frequently used frames of reference are approximately inertial frames. Are they comparable in the degree of approximation of the property 'to be inertial'? In other words, is it possible to measure the extent to which a frame deviates from the perfect regime of Galilean motion, so as to render the inertial property a *measurable* quantity? Below we give a positive answer to this question.

It transpires that the conventional definition of inertial frames, proposed by Neumann and Lange, is inappropriate for handling this problem. However, the proper acceleration of a given frame can be measured indirectly if the so-called apparent forces are taken into account. This issue is addressed in section 3. In contrast, an alternative definition of inertial frames based on the notion of unstable equilibrium [1] turns out to be adaptable to the desired measurements in a direct way. We offer the general principle of design of devices for making such measurements in section 4.

This discussion, apart from its pedagogical value, may be of utility in experiments on the detection of gravitational waves. It is anticipated that the Advanced LIGO [2], the laser interferometer detector which should be operational in 2014, will detect gravitational waves from bursts of highly relativistic objects such as supernovae. However, it may well be that gravitational waves from lower level sources are too feeble to be detected by the most sensitive operational instruments, as well as the proposed laser interferometer space antenna, LISA, consisting of three spacecraft in solar orbit. We then must take care to increase still further the measurement precision. Note that a platform for the detector—not only terrestrial but also spaceship-mounted—undergoes perturbations. To be specific, satellite-mounted detectors undergo numerous perturbations due to flows of cosmic waste, solar wind and irregularities in the Earth's rotation about its axis, not to mention minor accelerations owing to the Earth's orbit about the Sun, the Sun's orbit in the galaxy and the galaxy's motion in the Virgo Cluster. For example, a body with ballistic coefficient $C_B = 0.1$ m kg^{-1} separated from the Sun by one astronomical unit d_0 is exposed to radiation pressure $P \approx 1.4 \times 10^3$ W m^{-2}, which yields an acceleration of 0.5×10^{-7} g [3]. For a body separated from the Sun by a distance d, this value is multiplied by the factor $(d_0/d)^2$. To control the inertial property of the platform it is necessary to ensure that perturbations do not exceed some threshold. Unstable systems provide a useful check on whether this condition is fulfilled.

This paper is intended for advanced undergraduate and graduate students in high energy physics and relativity. Our aim is twofold: (i) to provide a deeper insight into the essentials of classical dynamics, and (ii) to give impetus to ingenious young people to devise new clever, useful and highly sensitive tools for measuring the inertial property following the pattern outlined in the present discussion.

2. The conventional treatment of inertial frames

Before going into experimental procedures aimed at rendering the inertial property a measurable quantity, effort must be made to refine the very notion of inertial frames of reference.

We first become aware of this notion in middle school where we gain an impression of inertial frames by a series of simple comparative examples: a carriage which is affected by pits and bumps versus a car which is moving gently along a smooth horizontal highway, a revolving carousel versus a chamber in a free fall, without rotation, etc. In the studentship season, we return to this notion at a higher level when we ponder over the question: 'Why did Newton accentuate the statement that a free body continues in its state of rest or uniform motion in a straight line as a separate law, Newton's first law, even though the state of non-accelerating motion is an obvious consequence of Newton's second law in the absence of external forces? Doesn't the equation of motion $m\ddot{\mathbf{x}} = \mathbf{0}$ imply that $\mathbf{x}(t) = \mathbf{x}_0 + \mathbf{v}t$, where $\mathbf{v} =$ const?' At this juncture, our tutor reminds us that while a free body indeed moves at a constant rate with respect to inertial frames, the term 'inertial' is yet to be explained. Newton's first law is not a law at all. This is a statement to introduce the notion of inertial frames. Making this statement, Newton gave an implicit definition of the type of frames with respect to which the laws of mechanics are to be formulated. Authoritative texts tell us: 'There is at least one frame of reference in which free particles move uniformly along straight lines. Every frame which has a uniform motion of translation relative to this frame is also an inertial frame' [4, 5]. The Springer *Encyclopedia of Physics* [6] gives just this definition of inertial frames.

Based on this argument, we envision that *free* particles move *uniformly* along *straight* lines in *inertial* frames assuming tacitly that we have appropriate criteria for deciding whether a given particle is 'free', its motion is 'uniform' and its path is 'rectilinear'. Strictly speaking, none of these qualities are possible to verify independently of each other when we have no prior knowledge that the frame under examination is inertial.

The naive idea that 'a particle is almost free if it is well off other particles' is valid until we recall *self-interaction*. Any particle is a source of some field which acts back on this particle. Does Newton's first law govern self-interacting particles? Experiment seems to point that sufficiently isolated bodies behave as free Galilean objects. However, it is not inconceivable that the current observations are insufficiently purposeful to grasp an anomaly in the motion of self-interacting particles. It has long been known that the Maxwell–Lorentz electrodynamics involves the so-called *self-accelerated* solutions (see e.g. [7, 8]): a free classical charged particle moves with the exponentially increasing four-acceleration squared,

$$a^2(s) = a^2(0)\exp(2s/\tau_0), \tag{1}$$

where s is the proper time, $a^2(0)$ is the initial acceleration squared and $\tau_0 = 2e^2/3mc^3$ is a characteristic time interval, where $\tau_0 \approx 6 \times 10^{-24}$ s if we choose e and m to be the charge and mass of a real electron. Note that equation (1) is consistent with Newton's first law. Indeed, if the motion was uniform before the instant $s = 0$, then $a^2(0) = 0$, and hence the motion is uniform for all time. Many people think of the self-accelerated solution (1) as an unphysical phenomenon because a charge continually accelerates and continually radiates. This may seem contrary to energy conservation. However, the reader may consult [8] to see that the energy violation is only apparent. The key idea is that self-interaction rearranges the initial mechanical and electromagnetic degrees of freedom into two new entities: *dressed particles* and *radiation*. The energy of a dressed particle is indefinite: increasing velocity need not be accomplished by increasing energy. A diligent student will verify that the energy of a dressed particle executing self-accelerated motion steadily decreases, which exactly compensates the increase in energy of the electromagnetic field emitted [9].

The notion of rectilinearity can be modelled in geometrical optics with the help of light rays. However, light propagates along a straight line only in inertial frames. Furthermore, the notion of uniform motion cannot be defined unless an inertial frame of reference is fixed because the time rate that reads our clocks is sensitive to the type of frame that has been chosen. This leads to a vicious circle: a notion $\mathcal{A}$ is formulated in terms of $\mathcal{B}$ and $\mathcal{C}$ whose own sense is intangible unless they are formulated in terms of $\mathcal{A}$.

It is widely believed that such circularity is inevitable in any physical discipline. According to this view, to define the fundamental notions individually and verify them separately is ill-advised but their system analysis in the unity would be a distinct possibility. To illustrate, the exposition of electrodynamics must not begin with complete definitions because 'the latter will be derived from their interrelation through the basic equations of the theory, which can be tested by experiment' [10]. A refined version of this teaching is: 'Here and elsewhere in science, as stressed not least by Henri Poincaré, that view is out of date which used to say, *Define your terms before you proceed*. All the laws and theories of physics, including the Lorentz force law, have this deep and subtle character, that they both define the concepts they use (here **B** and **E**) and make statements about these concepts. Contrariwise, the absence of some body of theory, law, and principle deprives one of the means properly to define or even to use concepts' [11].

The conventional view of the notions of 'uniform motion', 'straight line', 'free particle' and 'inertial frame' had formed at the turn of the 19th century (for historical details and further references see [12]). It was Lange who coined the expression *inertial frame* [13], which has since become standard. The idea of inertial frames was inspired by a proposal that Neumann made in his habilitation address [14], concerning time. Neumann realized that the notion of uniform motion is empty of content unless we know what is meant by 'equally long time intervals'. To define equal intervals of time he invoked the concept of free particles. Neumann reasoned that *two* free particles move in such a manner that equal path distances of the one always correspond to equal path distances of the other. The motion of one particle can be taken as defining equal units of time. This convention defines the *standard* time rate t, namely the rate which appears in Newton's second law.

The next step made by Lange [13] is to show that, for three or fewer particles, the rectilinearity of Newton's first law has no physical content, because whatever the motion of those particles, it is always possible to choose a coordinate system in which their trajectories are rectilinear. The claim that they are moving along straight lines is a matter of convention. On the other hand, relative to a coordinate system in which three free particles, projected from a single point, move in straight non-coplanar lines and travel mutually proportional distances, the motion of any fourth free particle will be rectilinear and uniform. (Lange's intricate line of reasoning is recast in an easy-to-read form in [15].) Lange called this coordinate system the inertial frame of reference.

The use of the notion of free particles to define the standard time rate and inertial frames rests heavily on the dynamical laws. Conceptually, the Neumann–Lange method is very sophisticated because the most fundamental elements of the scheme are defined in terms of themselves (for an extended discussion of this issue see [12]). Therein lies a weak point of the method: at least one fundamental element eludes definition. It may be worth emphasizing that the undefined element, the concept of free particles, is the least elementary if we keep in mind the self-interaction problem.

3. The apparent forces

Neumann–Lange's trajectory-based concept did not acquire experimental utility. All modern timekeeping methods use periodic processes rather than freely moving particles. The closest fit to the standard time rate is by atomic clocks. As to the experimental measurement of the inertial property of different frames, there is little point in thinking of how Lange's construction of four free particles can be adaptable for this purpose.

However, there exists an indirect way of measuring deviations of a frame of reference from the inertial state based on the phenomenon of *weight* experienced by a test mass that resides in this frame. All available accelerometers are devices that measure this kind of apparent force—weight per unit of test mass. An accelerometer senses the proper acceleration of this device, and hence the proper acceleration of the frame in which this accelerometer is rigidly fixed.

We begin with a brief overview of the design and performance of general purpose accelerometers (for an extended discussion see [16]). Electromechanical accelerometers are a commonly used tool in automotive, biomedical, industrial and numerous consumer applications since it is crucial for safety, measurement and control. The essential principle of design of a typical accelerometer reads: it behaves as a damped mass on a spring. When the accelerometer experiences an acceleration, the proof mass is displaced to the position that the spring is able to deform. The displacement q is then measured to give the acceleration a. Piezoelectric and capacitive components are widely used to convert the mechanical motion of the proof mass into an electrical signal.

Most common acceleration and vibration measurements are simple in nature, being either of compressional or torsional types. To perform them requires a single-axis system whose mechanical component is governed by

$$\ddot{q} + \nu\dot{q} + \omega_0^2 q = a. \tag{2}$$

Here, q is the proof mass displacement, ν is the damping factor and $\omega_0 = \sqrt{k/m}$ is the natural resonant frequency, with k and m being, respectively, the spring constant and the mass of the proof mass. (In order for more complex measurements related to, say, a combination of compressional, torsional and transverse vibrations to be made, we need a three-axis accelerometer whose behaviour is governed by a set of three ordinary second-order differential equations rather than a single equation.) The solution to equation (2)

$$q(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathrm{d}\omega \int_{-\infty}^{\infty} \frac{\mathrm{e}^{\mathrm{i}\omega(t-\tau)} a(\tau)\,\mathrm{d}\tau}{-\omega^2 + \mathrm{i}\omega\nu + \omega_0^2} \tag{3}$$

involves a major part of information about the accelerometer characteristics: sensitivity, frequency range, dynamic range and minimum detectable acceleration.

Note that m should be big enough to make the device quite immune to the mechanical noise due to Brownian motion of the proof mass. Here, we do not consider the electric chain noise which, however, may be dominant in such systems. On the other hand, m has to be small enough, for a given ν, to approach critical damping $\omega_0^2 = \nu^2/4$, which ensures maximum bandwidth. Piezoelectric accelerometers are unmatched in terms of their upper frequency range. Capacitive accelerometers excel in sensitivity and high performance in the low frequency range.

Mention should be made of kinematic accelerometers, which are based on timing the passage of an unconstrained proof mass between points marked on the accelerated base. These types of accelerometers find use as an absolute instrument in gravimetry.

Three-axis electrostatic accelerometers, exhibiting ultra-high sensitivities compatible with femto-g resolution [3, 17], are of primary concern to space applications such as the equivalence principle tests. An electrostatic accelerometer measures the voltage between charged electrodes in terms of force required to sustain a movable electrode at a given separation from affixed electrodes. The device is completed with a feedback system. An electrode of known mass and area is mounted on a light pivoted arm for moving relative to the fixed electrodes. The nominal gap between the pivoted and fixed electrodes is maintained by means of a force balancing servo system capable of varying the electrode potential in response to signals from a pick-off that senses relative changes in the gaps.

It is interesting that the inertial property of terrestrial frames lends itself to control much better than that of spaceship-mounted platforms. Gravimeters, the accelerometers that measure tiny changes within the Earth's surface gravity, are typically designed to be much more sensitive than usual general purpose accelerometers. The most accurate relative gravimeters are superconducting gravimeters, which achieve sensitivities of $10^{-12}\ g$. This is due to the fact that the basic constraints on sensitivity of accelerometers can be largely removed when a stable constant background acceleration is available, as happens with gravimetry.

It is possible to detect the proper acceleration of a non-inertial frame of reference which rotates with constant angular velocity $\mathbf{\Omega}$ about a fixed axis by taking into account the Coriolis force $-2m\mathbf{\Omega} \times \mathbf{v}$ and centrifugal force $-m\mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r})$, two further kinds of apparent forces which exert on a test particle of mass m in this frame. Here, $\mathbf{r}$ and $\mathbf{v}$ are, respectively, position vector, drawn from the origin on the axis of rotation, and velocity of the particle which is measured with respect to this rotating frame. A well-known case where the Coriolis force manifests itself as a measurable effect relates to the Foucault pendulum experiment. If a pendulum is set at the north pole, it must swing in a fixed plane while the Earth rotates beneath it. An observer on the Earth will then see that the plane of oscillation rotates with angular velocity $\mathbf{\Omega}$. In general, the apparent forces in rotating frames can be measured through the use of gyroscopes.

There exists the possibility of testing much more subtle phenomena predicted by general relativity, the geodetic and frame-dragging effects, by means of gyroscopes in the Earth's orbit. Indeed, the Gravity Probe B (GP-B) mission was designed to test whether there would be tiny

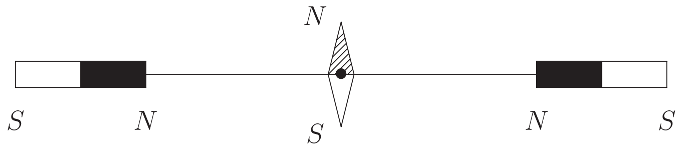

Figure 1. Magnetic needle in the state of unstable equilibrium.

changes in the directions of spin of four cryogenic gyroscopes contained in a satellite, circling the Earth in a polar orbit, in response to spacetime warping by the presence of the Earth and local spacetime dragging by the Earth's rotation about its axis [18]. The idea of the GP-B experiment is quite simple. A telescope is rigidly connected to the gyroscope housing. The telescope always points to a remote guide star, IM Pegasi, which provides the experiment's frame of reference in space. Initially, the gyroscopes' spin axes are aligned with this guide star. The gyroscopes, made up of fused quartz balls coated with superconducting niobium, rotate up to 5000 times min^{-1}. Each ball produces a magnetic field, so that changes in their orientations relative to the guide star could be detected. Analysis of the data from all four gyroscopes, gained as the spacecraft made over 5000 orbits around the Earth, results in a geodetic drift rate of -6601.8 ± 18.3 mas yr^{-1} and a frame-dragging drift rate of -37.2 ± 7.2 mas yr^{-1} to be compared with the theoretical predictions of -6606.1 and -39.2 mas yr^{-1}, respectively ('mas' is milliarc-second; 1 mas $= 4.848 \times 10^{-9}$ rad). Thus, GP-B provides measurements of the geodetic and frame-dragging effects at an accuracy of 0.28% and 19%, respectively [18].

4. Another way of looking at inertial frames

There exists an alternative concept of inertial frames [1]. A key idea is to use the notion of unstable equilibrium. Intuition suggests that states of unstable equilibrium of any physical system can be maintained only in inertial frames of reference because shocks and blows associated with accelerated motions of non-inertial frames prevent unstable systems from being balanced. This gives a simple operational criterion for distinguishing between inertial and non-inertial frames based on the capability of inertial frames for preserving unstable equilibria. An arrangement for checking whether the frame is inertial is shown in figure 1. A magnetic needle is installed halfway between north poles of two identical static magnets on the axis along which the magnets are lined up. A state of unstable equilibrium is attained when the needle is perpendicular to this axis. The magnets are mounted rigidly to the laboratory platform, and their separation is fixed. The system must be shielded by a protective metal screen and contained in a cryostat. A minute perturbation will suffice for the needle to swing through $+90°$ or $-90°$, so that its resulting direction is either aligned with or opposed to the magnet axis. The swing signals that a deviation from the state of Galilean motion occurs[3]. A triplet of such arrangements, set along the perpendicular axes, is sensitive to every small perturbation, including slow rotations of the frame.

This criterion is suitable for identifying not only idealized inertial frames but also real approximately inertial frames, which provides us with a simple and natural way for making the inertial property a measured quantity. To do this a testing system is required which is stable against infinitesimal perturbations but unstable against finite perturbations whose magnitude is

[3] An electrostatic accelerometer, minus the feedback system, may well be considered as an alternative to this arrangement. Indeed, by Earnshaw's theorem, the charged proof mass cannot be held in a stable equilibrium by electrostatic forces alone.

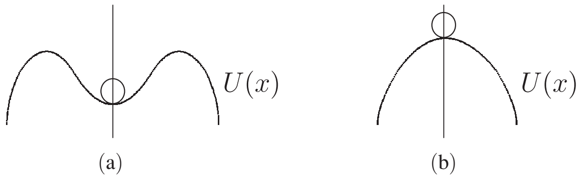

Figure 2. Two kinds of instability: (a) a particle in an equilibrium state, which is unstable against finite perturbations and (b) a particle in an absolutely unstable equilibrium.

Figure 3. A system in unstable and neutral equilibrium: a schematic representation of the key phenomena underlying the two concepts of inertial frames.

above some threshold. Figure 2 illustrates these two kinds of instabilities. We see a particle in two different potentials $U(x)$. One of them (left) affords a stable equilibrium state which can be violated when perturbations are greater than the depth of the potential pit while the other (right) refers to an absolutely unstable equilibrium. Given the former system as the testing device, the inertial property can be expressed in terms of the threshold value of perturbation. Indeed, for the potential of the form $U(x) = \frac{1}{2}\mu x^2 - \frac{1}{4}\lambda x^4$, displayed in the left plot, the threshold is $\Delta E = \frac{1}{4}\mu^2/\lambda$. This quantity may be used to express the range of accuracy within which the given frame is held to be inertial. Likewise, for the testing system shown in figure 1, use can be made of the frictional bond (static friction) on the axle of the magnetic needle. The magnetic needle swings if perturbations are large enough to overcome the frictional bond. Thus, the friction threshold of the device corresponds to the maximum allowable deviation of the frame from the Galilean regime of motion. If we take an array of devices whose operations relate to friction thresholds assuming different values, this would provide a way for making the inertial property a continuous quantity.

It is interesting that the conventional concept of inertial frames is readily translated into the language of equilibrium states: if a free particle is at rest in a particular inertial frame, then this particle may be regarded as being in the state of *neutral* equilibrium. Consider a potential $U(x)$, and suppose that $U(x)$ has a local maximum at $x = x^*$ and a plateau at $x^{**} \leqslant x \leqslant x^{***}$. These two agents for revealing inertial frames are shown in figure 3. Does every particle coming to $x = x^*$ attain the rest state? No. The extremum of $U(x)$ provides only the condition which is necessary for this to occur. The sufficient condition is that the velocity of the particle at $x = x^*$ is vanishing in some inertial frame. On the other hand, if the coordinates x are subjected to a nonlinear transformation, $x = x(\bar{x}, t)$, which is another way of stating that the inertial frame is changed for a non-inertial one, then the possibility of forming unstable equilibrium states disappears. Thus, the notion of unstable equilibrium, essential for this definition of inertial frames, is itself formulated in terms of inertial frames, which makes the definition circular. Is there a loop-hole which would allow us to unravel this tangle of notions?

Consider a Lagrangian system governed by the principle of least action:

$$\delta S = 0, \tag{4}$$

where S is the action of the system. For simplicity, we restrict our consideration to the case that the configuration space of the system is one dimensional; everything we say can readily be extended to higher dimensions. We interpret a solution to equation (4) as an equilibrium state of the system at $x = x^*$ if the history is depicted by a straight world line that issues out of the initial point (t_0, x^*). The existence of such states is due to the fact that we are dealing with inertial frames. Indeed, a nonlinear coordinate transformation $x = x(\bar{x}, t)$, associated with going to a non-inertial frame, would distort straight world lines. However, if our concern is with the general notion of the unstable state, no matter whether or not such states are balanced, then it is possible to explicate this notion in a coordinate-free form. It is well known from the calculus of variation that a solution $x(t)$ is unstable if

$$\frac{\delta^2 S}{\delta x^2} < 0. \tag{5}$$

The instability of $x(t)$ means that this solution depends heavily on the initial position x^* of the system:

$$\frac{\partial x(t; x^*)}{\partial x^*} \sim \exp(t/\Delta), \qquad t \gg \Delta. \tag{6}$$

Here, Δ stands for a characteristic time interval.

Perform a smooth coordinate transformation, the so-called diffeomorphism, $x = x(\bar{x}, t)$. Then, equation (4) implies

$$\frac{\delta S}{\delta \bar{x}} = \frac{\delta S}{\delta x}\frac{\partial x}{\partial \bar{x}} = 0. \tag{7}$$

It follows that

$$\frac{\delta^2 S}{\delta \bar{x}^2} = \frac{\delta^2 S}{\delta x^2}\left(\frac{\partial x}{\partial \bar{x}}\right)^2 + \frac{\delta S}{\delta x}\frac{\partial^2 x}{\partial \bar{x}^2} = \frac{\delta^2 S}{\delta x^2}\left(\frac{\partial x}{\partial \bar{x}}\right)^2 < 0, \tag{8}$$

where the second relationship is obtained through the use of equation (5). We thus see that both Hamilton's principle, equation (4), and the instability condition, equation (5), remain unchanged under smooth coordinate transformations. The notion of unstable states is diffeomorphism invariant. In other words, this notion can be defined in a coordinate-free manner.

Let the given Lagrangian system possess unstable states. Some of them could be promoted to unstable equilibrium states. With this aim in mind, we have to find such coordinates (global Cartesian coordinates) and time rate (standard time rate) in terms of which the selected states are represented by straight world lines. It remains to see whether self-interacting objects (or, more precisely, dressed particles) can be assembled into both stable and unstable bound states. If we are lucky, this procedure will be adapted for identifying inertial frames.

Devices such as the one in figure 1 may appear hardly practicable because they are valid for one occasion only, and it is not clear whether our line of reasoning can be modified to apply to permanent monitoring of the inertial property. Furthermore, if we have no prior knowledge of the frame, what is the reason to hope that we will succeed in balancing the magnetic needle? Note, however, that our main concern here is with the most basic points of the subject (the proposed alternative to the conventional definition of inertial frames, and the principle of operation of devices in unstable equilibrium) rather than details of the measurement procedures. There are many varieties of unstable systems, to mention just three: supercooled vapour, superheated liquid and an excited laser medium. With this hint in mind the ingenious reader is invited to make his or her own contrivances, following the above line of attack, to yield more useful instruments.

A few words are in order concerning the very idea of unstable equilibrium on the level of fundamental theory. It is clear that thermal and, even at zero temperature, quantum fluctuations must eventually disrupt any system which is in a truly unstable equilibrium, even if the system exists in an inertial frame. There is an irreducible limit to how finely any device could quantify the inertial property of a frame. In the long run, quantum fluctuations in the geometry of spacetime will put an end to the validity of the concept of inertial frames *concurrently* with the existence of unstable equilibrium states. From this, we might reason that these fundamental notions, the inertial frame and the unstable equilibrium state, are deeply intertwined.

5. Summary

The conventional paradigm of inertial frames is based on Neumann and Lange's idea of taking, as the starting point, the notion of free particles. Thus, the identification of inertial frames derives from the identification of free particles. Conceptually, this scheme is very sophisticated because the fundamental notions are defined in terms of themselves. Furthermore, the Neumann–Lange constructions are of no use for measuring the inertial property of approximately inertial frames of reference.

The definition of inertial frames based on the notion of unstable equilibrium is well suited to the operational identification of inertial frames. Systems in unstable equilibrium are capable of detecting the slightest deviations of the frame from the perfect Galilean state of motion. With an array of devices in unstable equilibrium, it is possible to render the inertial property of a frame a measurable quantity. In fact, we are dealing with a new type of accelerometer. Note that conventional accelerometry is successfully employed on condition that an extra inertial frame of reference is available. For example, the gyroscope precession in the GP-B experiment was measured with respect to a distant inertial frame associated with a guide star, as discussed in section 3. Accelerometers of the new type, contrastingly, dispense with the need for auxiliary frames of reference. These instruments will find use as soon as the ultimate accuracy will be required.

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