Foundations of Physics Letters, Vol. 19, No. 1, February 2006 (©2006)
DOI: 10.1007/s10702-006-1847-y

HELMHOLTZ THEOREM AND THE V-GAUGE IN THE PROBLEM OF SUPERLUMINAL AND INSTANTANEOUS SIGNALS IN CLASSICAL ELECTRODYNAMICS

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Received September 24, 2005

In this work we substantiate the applying of the Helmholtz vector decomposition theorem (H-theorem) to vector fields in classical electrodynamics. Using the H-theorem, within the framework of the two-parameter Lorentz-like gauge (so called *v-gauge*), we show that two kinds of magnetic vector potentials exist: one of them (solenoidal) can act exclusively with the velocity of light c and the other one (irrotational) with an arbitrary finite velocity v (including a velocity *more* than c) . We show also that the irrotational component of the electric field has a physical meaning and can propagate exclusively *instantaneously.*

Key words: Helmholtz theorem, v-gauge, electromagnetic potentials, electromagnetic waves.

1. INTRODUCTION

Lately the use of a two-parameter Lorentz-like gauge (so-called *v-gauge*, see, e.g., [1-5]) in classical electrodynamics gained popularity among physicists. Most likely one can explain this by attempting to provide, from the classical electrodynamics point of view an explanation of superluminal signals detected in a series of well-known experiments, performed at Cologne [6], Berkeley [7], Florence [8] and Viena [9], experiments by Tittel *et al.* [10] which revealed that evanescent waves (in undersized waveguides, e.g.) seem to spread with a superluminal group velocity. For example, in recent experiments by Mugnai *et al.* [11] superluminal behavior in the propagation of microwaves (centimeter wavelenth) over much longer distances (tens of centimeters) at a speed 7% faster than c was reported.

For example, in the recent work [5] by using the two-parameter Lorentz-like gauge (*v-gauge* [1-4]) and using the Helmholtz theorem it was shown that within the framework of classical electrodynamics the instantaneous action at a distance can exist (scalar potential acts *instantaneously* while the vector potential propagates at the speed of light) that implicitly confirms results of the works [12-14] (in these works the possibility of the existence of instantaneous action at a distance was rationalized out of the framework of the *v-gauge* theory). However the author of [5] does not substantiate the defensibility of the use of the Helmholtz vector decomposition theorem for time-dependent vector fields: The point is that recently J. A. Heras [15] showed that there is an inconsistent mathematical procedure here, which is due to the common misconception that the standard Helmholtz theorem [17] (which allows us to write $\mathbf{E} = \mathbf{E}_i + \mathbf{E}_s$, where $\mathbf{E}_i$ and $\mathbf{E}_s$ are irrotational and solenoidal components of the vector $\mathbf{E}$) can be applied to retarded (time-dependent) vector fields. In other words, when one introduces the time dependence into a vector field $\mathbf{E}$ and requires a decomposition of $\mathbf{E}$ into integral components one *must* prescribe the propagator. For electric and magnetic fields obeying Maxwell's equations, the causal propagator is the retarded Green function $D_R(x - x')$, where x is 4 variables (x, y, z, t). Thus, by Heras [15], the Helmholtz theorem for time-dependent vector fields must be formulated (using Heras' notation with $c = 1$) as follows: A time-dependent (retarded) vector field $\mathbf{E}(x)$ vanishing at spatial infinity is decomposed into *three* components *irrotational*, *solenoidal* and *temporal* one: $\mathbf{E} = \tilde{\mathbf{E}}_i + \tilde{\mathbf{E}}_s + \tilde{\mathbf{E}}_t$, where

$$\tilde{\mathbf{E}}_i(x) = -\nabla \int D_R(x - x')\nabla' \cdot \mathbf{E}(x')d^4x', \tag{1}$$

$$\tilde{\mathbf{E}}_s(x) = \nabla \times \int D_R(x - x')\nabla' \times \mathbf{E}(x')d^4x', \tag{2}$$

$$\tilde{\mathbf{E}}_t(x) = \frac{\partial}{\partial t} \int D_R(x - x')\frac{\partial \mathbf{E}(x')}{\partial t'}d^4x'. \tag{3}$$

In its standard formulation ($\mathbf{E} = \mathbf{E}_i + \mathbf{E}_s$) [17], Heras specifies (see footnote 2 in [15]) that Helmholtz theorem can consistently be applied to time-dependent vector fields *only* (!) when an instantaneous propagation for the fields is assumed.

Therefore results obtained in [5] *could be* incorrect if one takes into account the inferences of [15]. Nevertheless, taking into account this possible impropriety in [5], we have to note the following: the inferences of J. A. Heras [15] can be incorrect at least in the case of the time-dependent electric field written by means of scalar and vector potentials in the Coulomb gauge. It is obvious that for the electric field

$$\mathbf{E}(\mathbf{r}, t) = -\nabla\varphi(\mathbf{r}, t) - \frac{1}{c}\frac{\partial\mathbf{A}(\mathbf{r}, t)}{\partial t} \tag{4}$$

in this case an instantaneous propagation *is not* assumed because the field $\mathbf{E}$ in (4) can be a retarded solution of the wave equation

$$\nabla^2\mathbf{E} - \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} = 4\pi\left(\nabla\varrho + \frac{1}{c^2}\frac{\partial\mathbf{j}}{\partial t}\right). \tag{5}$$

Accordingly, it is clear that here although the electric field (4) can be *retarded*, it is decomposed into just *two* parts, one of which is *pure irrotational* and the other is *pure solenoidal*:

$$\mathbf{E} = \mathbf{E}_i + \mathbf{E}_s, \quad \mathbf{E}_i = -\nabla\varphi(\mathbf{r}, t), \quad \mathbf{E}_s = -\frac{1}{c}\frac{\partial\mathbf{A}(\mathbf{r}, t)}{\partial t} \tag{6}$$

(in the Coulomb gauge $\nabla\cdot\mathbf{A} = 0$). This alone shows that the inference of J.A. Heras [15] that a retarded field cannot be decomposed into *only* two parts (irrotational and solenoidal) can be insufficiently rigorous. Note also that in his recent work F. Rohrlich [16] has brought out clearly that the Standard Helmholtz theorem can be applied to time-dependent (retarded) vector fields.

2. TWO KINDS OF MAGNETIC VECTOR POTENTIAL

In our calculations we use the generalized gauge condition (the so-called *v-gauge*)

$$\nabla\cdot\mathbf{A} + \frac{c}{v^2}\frac{\partial\varphi}{\partial t} = 0, \tag{7}$$

the using of which in classical electrodynamics is already well-founded (see, e.g., [1-5]). Here v is some *arbitrary* velocity of propagation for electromagnetic potentials (and it is not necessarily that v has to be equal to c). In the Maxwell equations, if we express $\mathbf{E}$ and $\mathbf{B}$ through potentials, taking into account *v-gauge* (7) and after simple transformations we obtain

$$\nabla^2\varphi - \frac{1}{v^2}\frac{\partial^2\varphi}{\partial t^2} = -4\pi\varrho, \tag{8}$$

$$\nabla^2\mathbf{A} - \frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial^2 t} = \left(\frac{v^2 - c^2}{cv^2}\right)\nabla\frac{\partial\varphi}{\partial t} - \frac{4\pi}{c}\mathbf{j}. \tag{9}$$

Let us start from Eq. (9) which, taking into account Eq. (7), can be written as

$$\nabla^2\mathbf{A} - \frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial^2 t} + \left(\frac{v^2-c^2}{c^2}\right)\nabla(\nabla\cdot\mathbf{A}) = -\frac{4\pi}{c}\mathbf{j} \tag{10}$$

or, using the identity $\nabla(\nabla\cdot\mathbf{A}) = \nabla\times(\nabla\times\mathbf{A}) + \nabla^2\mathbf{A}$ and multiplying by c^2/v^2, we find

$$\nabla^2\mathbf{A} - \frac{1}{v^2}\frac{\partial^2\mathbf{A}}{\partial^2 t} + \left(\frac{v^2-c^2}{v^2}\right)\nabla\times(\nabla\times\mathbf{A}) = -\frac{4\pi c}{v^2}\mathbf{j}. \tag{11}$$

Now let the vectors $\mathbf{A}$ and $\mathbf{j}$ satisfy the conditions of the Helmholtz theorem. So

$$\mathbf{A}(\mathbf{r},t) = \mathbf{A}_s(\mathbf{r},t) + \mathbf{A}_i(\mathbf{r},t), \quad \text{and} \quad \mathbf{j}(\mathbf{r},t) = \mathbf{j}_s(\mathbf{r},t) + \mathbf{j}_i(\mathbf{r},t). \tag{12}$$

After substituting Eqs.(12) into (10) and (11), we have, respectively,

$$\nabla^2(\mathbf{A}_s+\mathbf{A}_i) - \frac{1}{c^2}\frac{\partial^2(\mathbf{A}_s+\mathbf{A}_i)}{\partial^2 t} + \left(\frac{v^2-c^2}{c^2}\right)\nabla(\nabla\cdot(\mathbf{A}_s+\mathbf{A}_i)) = -\frac{4\pi}{c}(\mathbf{j}_s+\mathbf{j}_i), \tag{13}$$

$$\nabla^2(\mathbf{A}_s+\mathbf{A}_i) - \frac{1}{v^2}\frac{\partial^2(\mathbf{A}_s+\mathbf{A}_i)}{\partial^2 t} + \left(\frac{v^2-c^2}{v^2}\right)\nabla\times(\nabla\times(\mathbf{A}_s+\mathbf{A}_i)) = -\frac{4\pi c}{v^2}(\mathbf{j}_s+\mathbf{j}_i). \tag{14}$$

By virtue of the uniqueness of the decomposition of vectors into solenoidal and irrotational parts (see [19], e.g.) one can equate solenoidal components of *lhs* and *rhs* of Eq.(13) and irrotational components of *lhs* and *rhs* of Eq. (14). The resulting equations are

$$\nabla^2\mathbf{A}_s - \frac{1}{c^2}\frac{\partial^2\mathbf{A}_s}{\partial t^2} = -\frac{4\pi}{c}\mathbf{j}_s, \tag{15}$$

$$\nabla^2\mathbf{A}_i - \frac{1}{v^2}\frac{\partial^2\mathbf{A}_i}{\partial t^2} = -\frac{4\pi c}{v^2}\mathbf{j}_i. \tag{16}$$

Thus one can see that two kinds of magnetic vector potential exist: one of which ($\mathbf{A}_s$) propagates exclusively with the velocity of light c and the other one with an arbitrary velocity v (including $v > c$). Note that for $v \to \infty$ the vector potential $\mathbf{A}_i$ vanishes within the framework of the conditions of the Helmholtz theorem (in accordance with the assertion of [5]). However one can see that $\mathbf{A}_i$ exists for $c < v < \infty$.

3. TWO KINDS OF ELECTRIC FIELD

Note, *however*, the following very important "feature": in the *v-gauge* the irrotational part of the electric field (4) can propagate instantaneously only!

Indeed, if we let the operator "−grad" act on Eq. (8), we obtain

$$\nabla^2 \mathbf{E}_\varphi - \frac{1}{v^2}\frac{\partial^2 \mathbf{E}_\varphi}{\partial t^2} = 4\pi \nabla \varrho, \tag{17}$$

where $\mathbf{E}_\varphi = -\nabla\varphi$ is a field produced exclusively by means of the electric potential from (4). Next we rewrite (4) in the form

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}_\varphi(\mathbf{r},t) - \frac{1}{c}\frac{\partial \mathbf{A}_i(\mathbf{r},t)}{\partial t} - \frac{1}{c}\frac{\partial \mathbf{A}_s(\mathbf{r},t)}{\partial t} \tag{18}$$

or

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}_i(\mathbf{r},t) + \mathbf{E}_s(\mathbf{r},t), \tag{19}$$

where, obviously,

$$\mathbf{E}_i(\mathbf{r},t) = \mathbf{E}_\varphi(\mathbf{r},t) - \frac{1}{c}\frac{\partial \mathbf{A}_i(\mathbf{r},t)}{\partial t}, \tag{20}$$

$$\mathbf{E}_s(\mathbf{r},t) = -\frac{1}{c}\frac{\partial \mathbf{A}_s(\mathbf{r},t)}{\partial t}. \tag{21}$$

Let us now act with the operator "$-\frac{1}{c}\frac{\partial}{\partial t}$" on Eq. (16):

$$\nabla^2\left\{-\frac{1}{c}\frac{\partial \mathbf{A}_i(\mathbf{r},t)}{\partial t}\right\} - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\left\{-\frac{1}{c}\frac{\partial \mathbf{A}_i(\mathbf{r},t)}{\partial t}\right\} = \frac{4\pi}{v^2}\frac{\partial \mathbf{j}_i}{\partial t}. \tag{22}$$

Finally, summing (22) and (17) and taking into account (20), we get

$$\nabla^2 \mathbf{E}_i - \frac{1}{v^2}\frac{\partial^2 \mathbf{E}_i}{\partial t^2} = 4\pi\left(\nabla\varrho + \frac{1}{v^2}\frac{\partial \mathbf{j}_i}{\partial t}\right). \tag{23}$$

It is obvious[1] that this expression reduces to $\nabla^2 \mathbf{E}_i = 4\pi\nabla\varrho$.

Correspondingly, for $\mathbf{E}_s$ we have

$$\nabla^2 \mathbf{E}_s - \frac{1}{c^2}\frac{\partial^2 \mathbf{E}_s}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial \mathbf{j}_s}{\partial t}. \tag{24}$$

[1]Applying the Helmholtz theorem to the Maxwell equation $\nabla\times\mathbf{B} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c}\mathbf{j}$, after time differentiation we obtain $\frac{\partial^2 \mathbf{E}_i}{\partial t^2} = -4\pi\frac{\partial \mathbf{j}_i}{\partial t}$.

Thus we see that the vector fields $\mathbf{E}_i$ and $\mathbf{E}_s$ are solutions of the *different* equations with $\mathbf{E}_i$-"wave" propagating *instantaneously* and $\mathbf{E}_s$-wave propagating with the velocity c respectively.

By virtue of the uniqueness of the decomposition of vectors into solenoidal and irrotational parts, the values of $\mathbf{E}_i$ and $\mathbf{E}_s$ cannot depend on a gauge. To verify this let us now construct the wave equation for the field $\mathbf{E}$ from the Maxwell equations

$$\operatorname{rot}\mathbf{B} = \frac{4\pi}{c}\mathbf{j} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}, \quad \operatorname{rot}\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}, \quad \operatorname{div}\mathbf{E} = 4\pi\varrho, \tag{25}$$

$$\nabla^2\mathbf{E} - \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} = 4\pi\left(\nabla\varrho + \frac{1}{c^2}\frac{\partial \mathbf{j}}{\partial t}\right). \tag{26}$$

Then, after applying the Helmholtz theorem to the vectors $\mathbf{E}$ and $\mathbf{j}$ in (26), we can equate solenoidal and irrotational parts of *lhs* and *rhs* of (26) respectively by virtue of the uniqueness of the decomposition of vectors in accordance with the Helmholtz theorem. The resultant equations are (see footnote 1):

$$\nabla^2\mathbf{E}_i - \frac{1}{c^2}\frac{\partial^2\mathbf{E}_i}{\partial t^2} = 4\pi\left(\nabla\varrho + \frac{1}{c^2}\frac{\partial \mathbf{j}_i}{\partial t}\right) \quad \Longrightarrow \quad \nabla^2\mathbf{E}_i = 4\pi\nabla\varrho, \tag{27}$$

$$\nabla^2\mathbf{E}_s - \frac{1}{c^2}\frac{\partial^2\mathbf{E}_s}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial \mathbf{j}_s}{\partial t}. \tag{28}$$

In that way the "paradox" that $\mathbf{E}_i$ can be *simultaneously* retarded and istantaneous, observed by Rohrlich (see Eqs. 3.12-3.17 in [18]), is resolved: one can see that $\mathbf{E}_i$ must be exclusively *instantaneous*.

Let us consider the case when exlusively $\mathbf{E}_i$ can be responsible for a signal transfer from one point charge q to the other point charge Q (or to some fixed point of observation).

Let us suppose the charge q is vibrating by means of some non-electrical force along the X-axis, then charge Q (or the fixed point of observation), lying at the same axis at some fixed distance from the charge q vibration centre, will obviously "know" that charge q is vibrating: in the observation point the value of the energy density w (which is a point function of $\mathbf{E}$ at the X-axis) will also oscillate.

Let us now analyse the equations for $\mathbf{E}_i$, $\mathbf{E}_s$, $\mathbf{j}_i$ and $\mathbf{j}_s$:

$$\nabla\cdot\mathbf{E}_i = 4\pi\varrho, \tag{29}$$

$$\frac{\partial \mathbf{E}_i}{\partial t} = -4\pi\mathbf{j}_i, \tag{30}$$

and, for solenoidal components,

$$\nabla \times \mathbf{E}_s = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}, \tag{31}$$

$$\nabla \times \mathbf{B} = \frac{1}{c}\frac{\partial \mathbf{E}_s}{\partial t} + \frac{4\pi}{c}\mathbf{j}_s. \tag{32}$$

From (31) and (32), we obtain the wave equations

$$\nabla^2 \mathbf{E}_s - \frac{1}{c^2}\frac{\partial^2 \mathbf{E}_s}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial \mathbf{j}_s}{\partial t}, \tag{33}$$

$$\nabla^2 \mathbf{B} - \frac{1}{c^2}\frac{\partial^2 \mathbf{B}}{\partial t^2} = -\frac{4\pi}{c}\nabla \times \mathbf{j}_s. \tag{34}$$

Here one can see that the solenoidal components of the electromagnetic field are in charge of the electromagnetic radiation with the derivatives of $\mathbf{j}_s$ as a source of these waves. Let us consider now the field created by a point charge with an arbitrary movement:

$$\mathbf{r}_q = \mathbf{r}_q(t), \tag{35}$$

$$\mathbf{v}_q = \mathbf{v}_q(t) = d\mathbf{r}_q(t)/dt. \tag{36}$$

The charge density and current density are given as

$$\varrho(\mathbf{r}, t) = q\delta(\mathbf{r} - \mathbf{r}_q(t)), \tag{37}$$

$$\mathbf{j}(\mathbf{r}, t) = q\mathbf{v}_q\delta(\mathbf{r} - \mathbf{r}_q(t)). \tag{38}$$

These quantities are not independent:

$$\frac{\partial \varrho}{\partial t} + \nabla \cdot \mathbf{j} = 0. \tag{39}$$

Let us find the irrotational and solenoidal components of the current density. From the Helmholtz theorem, we have

$$\mathbf{j}_i(\mathbf{r}, t) = -\frac{1}{4\pi}\nabla \int \frac{\nabla' \cdot \mathbf{j}(\mathbf{r}', t)}{|\mathbf{r} - \mathbf{r}'|} dV'. \tag{40}$$

Taking into account Eq. (39), we obtain

$$\mathbf{j}_i(\mathbf{r}, t) = \frac{1}{4\pi}\nabla \frac{\partial}{\partial t} \int \frac{\varrho(\mathbf{r}', t)}{|\mathbf{r} - \mathbf{r}'|} dV'. \tag{41}$$

Finally, substituting (37), we have

$$\mathbf{j}_i(\mathbf{r},t) = \frac{q}{4\pi}\nabla\frac{\partial}{\partial t}\frac{1}{|\mathbf{r}-\mathbf{r}_q(t)|}. \tag{42}$$

One can rewrite this expression in the form

$$\mathbf{j}_i(\mathbf{r},t) = -\frac{1}{4\pi}\frac{3\mathbf{n}(\tilde{\mathbf{j}}\cdot\mathbf{n})-\tilde{\mathbf{j}}}{|\mathbf{r}-\mathbf{r}_q(t)|^3}, \tag{43}$$

where

$$\tilde{\mathbf{j}} = q\mathbf{v}_q(t), \tag{44}$$

$$\mathbf{n} = \frac{\mathbf{r}-\mathbf{r}_q(t)}{|\mathbf{r}-\mathbf{r}_q(t)|}. \tag{45}$$

Pay attention to the similitude of Eq. (43) and the well-known expression for the electric field created by a dipole.

On the other hand, for the solenoidal component we have

$$\mathbf{j}_s(\mathbf{r},t) = \frac{1}{4\pi}\nabla\times\int\frac{\nabla'\times\mathbf{j}(\mathbf{r}',t)}{|\mathbf{r}-\mathbf{r}'|}dV'. \tag{46}$$

After some calculations, we get

$$\mathbf{j}_s(\mathbf{r},t) = q\mathbf{v}_q\delta(\mathbf{r}-\mathbf{r}_q(t)) + \frac{1}{4\pi}\frac{3\mathbf{n}(\tilde{\mathbf{j}}\cdot\mathbf{n})-\tilde{\mathbf{j}}}{|\mathbf{r}-\mathbf{r}_q(t)|^3}. \tag{47}$$

So, comparing (47) and (43), we conclude that

$$\mathbf{j}_s(\mathbf{r},t) = -\mathbf{j}_i(\mathbf{r},t) \tag{48}$$

at every point with the exception of the point of location of the charge.

The obtained expressions permit us to find the irrotational component $\mathbf{E}_i$ of the electric field created by the charge. Comparing (30) and (42), we obtain

$$\mathbf{E}_i = q\frac{\mathbf{r}-\mathbf{r}_q(t)}{|\mathbf{r}-\mathbf{r}_q(t)|^3}. \tag{49}$$

One can see that the field $\mathbf{E}_i$ is a Coulomb type field: it is conservative and has a spherical symmetry with respect to the instantaneous location of the charge. Besides the field $\mathbf{E}_i$ “moves” (changes) instantaneously everywhere in space together with the charge.

As an example, consider the case when the point charge q performs harmonic oscillations along the X-axis:

$$\mathbf{r}_q(t) = (A_0\sin\omega t)\mathbf{i}, \tag{50}$$

where $\mathbf{i}$ is the unit vector in the positive direcction of the X-axis. Using (42) and (48) we obtain for any point[2] on the X-axis ($\mathbf{r} = x\mathbf{i}$):

$$\mathbf{j}_i = -\mathbf{j}_s = -\frac{q}{4\pi}\frac{A_0\omega\cos\omega t}{|x - A_0\sin\omega t|}\mathbf{i}, \tag{51}$$

$$\mathbf{E}_i = q\frac{x - A_0\sin\omega t}{|x - A_0\sin\omega t|^3}\mathbf{i}. \tag{52}$$

So one can see that $\mathbf{E}_i$ is directed along the X-axis on the X-axis.

In order to determine the solenoidal component $\mathbf{E}_s$ we do the following. The field $\mathbf{E}$ created by the charge has to be periodic and consequently we can develop its solenoidal and irrotational components in the Fourier series:

$$\mathbf{E}_i(\mathbf{r}, t) = \sum_{n=0}^{\infty}\mathbf{E}_{in}e^{-in(\mathbf{k}\cdot\mathbf{r}-\omega t)}, \tag{53}$$

$$\mathbf{E}_s(\mathbf{r}, t) = \sum_{n=0}^{\infty}\mathbf{E}_{sn}e^{-in(\mathbf{k}\cdot\mathbf{r}-\omega t)}. \tag{54}$$

From the properties of these fields,

$$\nabla \times \mathbf{E}_i = 0, \tag{55}$$

$$\nabla \cdot \mathbf{E}_s = 0, \tag{56}$$

we obtain, for every n,

$$\mathbf{k} \times \mathbf{E}_{in} = 0, \tag{57}$$

$$\mathbf{k} \cdot \mathbf{E}_{sn} = 0. \tag{58}$$

The last equations mean that the vectors $\mathbf{E}_i$ and $\mathbf{E}_s$ must be mutually perpendicular everywhere in space and thus $\mathbf{E}_s$ must be perpendicular to the X-axis in every point of the X-axis ($\mathbf{E}_i$ (Eq. (52) is collinear to X-axis).

On account of the symmetry of the problem and because of $\mathbf{E} = \mathbf{E}_i + \mathbf{E}_s$, $\mathbf{E}_s$ must be equal to *zero* along of the X-axis. It can mean solely the following: The irrotational component of the electric field has a physical meaning and in some cases is charged with the instantaneous energy and momentum transmission.

So we have made sure that the irrotational component of the electric field has a physical meaning and in some cases, obviously, *it* is solely in charge of the energy and momentum transmissions which, evidently, have to be instantaneous in this case (see Eqs. (23), (27)

[2]Except for the location point of q, of course.

and footnote 1). It is obvious also that this field cannot be directly obtained from the well-known expression for the electric field created by an arbitrarily moving charge,

$$\mathbf{E}(\mathbf{r},t) = q\left\{\frac{(\mathbf{R}-R\frac{\mathbf{V}}{c})(1-\frac{V^2}{c^2})}{(R-\mathbf{R}\frac{\mathbf{V}}{c})^3}\right\}_{t_0} + q\left\{\frac{[\mathbf{R}\times[(\mathbf{R}-R\frac{\mathbf{V}}{c})\times\frac{\dot{\mathbf{V}}}{c^2}]]}{(R-\mathbf{R}\frac{\mathbf{V}}{c})^3}\right\}_{t_0}, \tag{59}$$

where $\mathbf{V} = \mathbf{V}(t_0)$ is the velocity of the charge q, $t_0 = t - R/c$, $R = |\mathbf{r} - \mathbf{r}_q(t_0)|$, because the full field $\mathbf{E}$ in (59) was obtained from the Liénard-Wiechert potentials taking into account the retardation, and our field $\mathbf{E} = \mathbf{E}_i$ at the X-axis *is not retarded* in accordance with Eqs. (23), (27). So at the X-axis, taking into account the *instantaneousness* of $\mathbf{E}_i$ we must put a velocity of the propagation of the field $\mathbf{E}$ $c = \infty$ and then we obtain Eq. (52).

4. CONCLUSION

One can see that the irrotational part $\mathbf{A}_i$ of the vector potential $\mathbf{A}$ and the scalar potential φ *can* propagate with an arbitrary *finite* velocity including a velocity *more* than c as well as instantaneously in the case of the scalar potential (we showed that for $v \to \infty$ the vector potential $\mathbf{A}_i$ vanishes within the framework of the conditions of the Helmholtz theorem).

In regards to the *irrotational* conponent of the electric field (see Eqs. (23) and (27)), it has a physical meaning and can propagate exclusively *instantaneously*. Therefore we can conclude that there are *two* mechanisms of the energy and momentum transmission in classical electrodynamics:

(1) the *retarded* one by means of a radiation ($\mathbf{E}_s$ and $\mathbf{B}$), see Eqs. (33), (34);

(2) the *instantaneous* one by means of the irrotational field $\mathbf{E}_i$.

Note that for the describing of an energy transfer in the second mechanism along the line of the interaction of two point charges the use of the Poynting vector concept makes no sense at all. Note also that $\mathbf{E}_i$ cannot have any functional relations with the magnetic field (see Eqs. (29)-(34)). Thus we see that field $\mathbf{E}_i$ although is materially existent, cannot participate in the phenomenon known as *electromagnetic wave*.

In view of the obvious spherical symmetry and the non-retardation of the field $\mathbf{E}_i$ (49) we would like to make a quotation of P.A.M. Dirac: "*As long as we are dealing only with transverse waves, we cannot bring in the Coulomb interactions between particles.*

To bring them in, we have to introduce longitudinal electromagnetic waves... We thus get a new version of the theory, in which the electron is always accompanied by the Coulomb field around it. Whenever an electron is emitted, the Coulomb field around it is simultaneously emitted, forming a kind of dressing for the electron. Similarly, when an electron is absorbed, the Coulomb field around it is simultaneosly absorbed. This is, of course, very sensible physically, but it also means a rather big departure from relativistic ideas. For, if you have a moving electron, then the Coulomb field around it is not spherically symmetrical[3]*. Yet it is the spherically symmetric Coulomb field that has to be emitted here together with the electron.*" [20]. So there is good reason to believe that exactly the field $\mathbf{E}_i$ can play a role of the spherically symmetric electric field, which is mentioned by Dirac, which always accompanies any point charge and it is not a generally accepted Coulomb field because it depends on time. We would like to name this field $\mathbf{E}_i$ "Dirac's field".

Everything described above can provide a theoretical rationale (within the framework of classical electrodynamics) of a series of well-known experiments [6-11] mentioned in the Introduction. Nevertheless it is significant that one can find a theoretical rationale of the existence of the superluminal interaction out of the framework of the Helmholtz theorem and *v-gauge*-theory in the review works [21] and [22] (see also the review [23]).

Finally, we can affirm that applying the Helmholtz theorem to classical ekectrodynamics allows us to conclude that in classical electrodynamics so called *instantaneous action at a distance* with the *infinite* velocity of interaction *can take place* as well as (within the framework of the *v-gauge*-theory) the superluminal action with a *finite* velocity of interaction.

Acknowledgements. We are grateful to V. Onoochin and Dr. V. Hnizdo for many stimulating discussions and critical comments.

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