Hilbert vs. exponential Kernel functionals for Nonlocal Means image filtering

J. I. de la Rosa Vargas, Senior Member, IEEE* †, J. J. Villa*, J. Cortez†, E. González*
and E. de la Rosa*
*Universidad Autónoma de Zacatecas, Unidad Académica de Ingeniería
Eléctrica, Av. López Velarde No. 801, Col. Centro, C.P. 98068, Zacatecas, Zacatecas.
†Instituto Tecnológico de Sonora, Campus Náinari, Depto. de Ingeniería Eléctrica
y Electrónica,Av. Antonio Caso No. 2266, Col. Villa ITSON, C.P. 85130, Cd. Obregón, Sonora.
Email: ismaelrv@ieee.org

***Abstract*—The present work introduces a new alternative to change the classical exponential kernel function used in Nonlocal Means (NLM) methods to deal with digital image filtering. The idea is based on the premise that making a good selection or estimation of the bandwidth parameter h is difficult and there are some other kernels which have another equivalent parameters to be selected into a more easiest way. A First method is obtained, when using an optimal manner proposed in nonparametric estimation literature to estimate h to tune the exponential kernel function. And a second proposed method, is to change the exponential function by the so called Hilbert function where one must to choose a parameter d. This Hilbert function is used for the first time in the NLM framework. Finally, the obtained filtering results reveals, that the NLM Hilbert kernel approach gives similar performance to other approaches according to recent reported results in literature, and the first proposed methodology is restricted by the estimation of h.**

I. Introduction

The image denoising methods are at the basis of digital image processing and still there are challenging problems to construct robust filters in applications where the noise is very disperse or where it is of multiplicative nature such as the speckle type noise. Nowadays, the nonlocal means (NL-means) filtering has been established as a reference tool, since this type of filters deals with the preservation of structure and objects into a digital image (introduced in the seminal work of Buades [2]). The "method noise" is the core of the mathematical analysis of such nonlocal means (NLM) filters where an error is defined as the difference between a true digital image and its denoised version.

The image filtering, is a particular task of restoration or recuperation approaches of an image to its original condition given a degraded image. The restoration passes by reverting the effects caused by a distortion function which must be estimated in most of the practical cases. In fact, the degradation characteristic is a crucial information and it must be supposed known or estimated during the inversion procedure. Typically, this is a point spread function which can be linked with the probability distribution of the noise contamination. Thus, a global image formulation model could be given by:

$$y = F(x) + n, \tag{1}$$

where, $F(x)$ is a functional that could take for instance, two forms: $F(x) = x$ and $F(x) = Hx$, being H a linear operator which models the image degradation (i.e. convolution). All variables presented along the text are, x: which represents an image to be estimated, y: represents the observed image with additive noise n and/or distorted by H, and $\widehat{x}$: is the estimator of x with respect to data y.

Since the introduction of the NLM approach proposed by Buades et al [2], [3], [4] for image filtering, several generalizations or alternatives have been proposed in the literature, going from hybrid approaches (NLM and Wavelets, NLM and Linear Transforms) [16], [26], [27], NL Variational Methods, NL anisotropic patches [19], the focusing on the calculations of an appropriate or optimal bandwidth [24], [25], the appropriate size or geometry of the patches or windows of analysis [8], and the change of the kernel or weighting function to be used [8], [13], [23], since the most of time the exponential function is used. Also, the notion of self-similarity or redundancy has been explored to construct methods to accelerate the NLM approach such as the pre-selection of the contributing neighborhoods based on average value and gradient, average and variance or higher-order statistical moments, cluster tree arrangement, and singular value decomposition. Also the computation of the distance measure between different neighborhoods can be optimized using the fast Fourier transform [8], [17] or a moving average filter.

The classical exponential kernel function is used in most of the cases as a weighting function in NLM methods to deal with digital image filtering as it can be seen in equation (7). However, the performance of the NLM depends on an optimal choice of the bandwidth parameter h, and other related parameters such as the size of the neighborhood, and the search region. The main idea in the proposition of the present work is based on the premise that making an optimal selection of the bandwidth parameter h is difficult (a problem interestingly solved in the work of Van De Ville [24], [25] by using the Stein's Unbiased Risk Estimate-SURE) and there are some other kernels which have another equivalent parameters to be set into a more easiest way [12], [14]. First, it is proposed the use of an optimal manner proposed in nonparametric literature to choose h [9], [10], [11], [21], [22]. Then, in a second proposition it is changed the exponential by the Hilbert function introduced by Devroye and Krzyzȧk in [12], and previously used in some other works [5], [6], [7]. It is the first time that the Hilbert function is used into the NLM framework, but it has been used in previous work for robust image filtering

978-1-4799-7436-8/15/$31.00 ©2015 IEEE