Bayesian Entropy Estimation: Applications in Robust Image Filtering

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Abstract

We introduce a new approach for image filtering in a Bayesian framework, in this case the probability density function (pdf) of the likelihood function is approximated using the concept of non-parametric or kernel estimation. The method is also based on generalized Gaussian Markov random fields (GGMRF), a class of Markov random fields which are used as prior information into the Bayesian rule, which principal objective is to eliminate those effects caused by the excessive smoothness on the reconstruction process of images which are rich in contours or edges. Accordingly to the hypothesis made for the present work, it is assumed a limited knowledge of the noise pdf, so the idea is to use a non-parametric estimator to estimate such a pdf and then apply the entropy to construct the cost function for the likelihood term. The previous idea leads to the construction of Maximum a posteriori (MAP) robust estimators, since the real systems are always exposed to continuous perturbations of unknown nature. Some promising results of three new MAP entropy estimators (MAPEE) for image filtering are presented, together with some partial concluding remarks.

1. Introduction

The image restoration approaches or recuperation of an image to its original condition given a degraded image, passes by reverting the effects caused by a distortion functional which in some cases is known or must be estimated. The basic idea in Bayesian estimation is to construct a Maximum a posteriori (MAP) of the modes or so called estimator of true images by using Markov random fields (MRF's). The evolution of the basic idea has caused the development of new algorithms which consider new models of contextual information which is led by the MRF's and the final aim is the restoration of real images. The idea is based on a robust scheme which could be adapted to reject outliers, tackling situations where noise is present in different forms during the acquisition process. In the case of classical MAP filters, usually the additive Gaussian noise is considered, however in some applications this noise is non-Gaussian [2] or unknown (with some partial knowledge). This is a source of information which imposes a key rule in the image processing context (the contextual or spatial information), that represents the likelihood function or correlation between the intensity values of a well specified neighborhood of pixels. Also, the modeling when using MRF's take into account such spatial interaction and it was introduced and formalized in [3] where it is shown the powerfulness of these statistical tools [4], [5], [6], [15], [22]. The image modeling in the context of the present paper lead us to assume a limited knowledge about the image noise pdf ($p(e) = p(y|x)$), so we propose to use the data itself to obtain a non-parametric entropy estimate (EE) of the log-likelihood pdf ($\widehat{p}_{n,h}(e)$) [7], [8], [9]. Then the log-likelihood will be optimized (e.g. minimized) together with a log-MRF to obtain the MAP image estimation.

A variety of applications in signal processing and instrumentation, are based in statistical modeling analysis. One of the most used is the linear regression model (simple or multi-variable in the case of images)

$$y_{i,j} = x_{i,j}^{\top}\theta_{i,j} + e_{i,j}, \text{ with } e \sim p(e), \tag{1}$$

where y represents the response (observed data, acquired data or explained variables), to x explicative variables for $i = 1, \ldots, N$ and $j = 1, \ldots, M$, and a system response parameterized by θ which is associated to data (y, x). In some applications θ are functional parameters which will be estimated by an identification procedure if x are known, but if θ are known and x are unknown, the estimation is made about x, or the estimation can be made for both cases (e.g. blind deconvolution). The e variables are random processes which are independent and identically distributed accordingly to $p(e)$. A natural extension of the linear regression

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