Convergence of Minimum-Entropy Robust Estimators: Applications in DSP and Instrumentation

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Abstract

In this paper we propose to continue in the same research line initiated by Pronzato and Thierry [13], [14], [15], recent works inspired in the minimum-entropy estimation have been published by De la Rosa and Fleury [2], [3] in the instrumentation framework. An statistical model has been established to represent some instrumental signals, similarly, some limited hypothesis over such a model have been made. In fact, we assume limited knowledge of the noise or external perturbations distribution that interact into the system. The use of robust estimators in such situations is very helpful, since the real systems are always exposed to continuous perturbations of unknown nature. Some applications where the last is true are: medical instrumentation, industrial processes, in telecommunications among others. Some results of new minimum-entropy estimators for linear and nonlinear models are presented, such results complement those presented by Pronzato and Thierry.

1. Introduction - State of the art

In industrial processes or in medical instrumentation, the signal acquisition for monitoring or data analysis are current tasks, for example, this could be made to assure quality of the production processes or for medical patient monitoring. Any other application on industry, on medicine, or other discipline could be concerned with a classical problem (it has been treated by decades) of data acquisition, that means the processing of signals contaminated by correlated or external perturbations which affect the system under employment. The perturbations are also called noise and this is due to rounding errors, some failures in the measurement equipment, etc.

A classical estimator used in statistics is the Maximum Likelihood (ML) estimator, this procedure minimizes the entropy of the empirical probability density function of the system errors or residuals ($\wp(\boldsymbol{e})$), when such a distribution is unknown the ML estimator could be suboptimal, since the residual distribution considered can be inadequate. In such case, in the section 3 is introduced a recent robust estimator which is based in the entropy minimization of an estimated version of the probability density function of residuals ($\widehat{\wp}_{n,h}(\boldsymbol{e})$) using nonparametric estimation. This estimated version is attained thanks to the residuals empirical distribution. In fact, the proposed nonparametric procedure is lead by the classical kernel estimators.

The principal apport of this work is presented in sections 4 and 5, where a comparison of the different Minimum-Entropy estimators (MEE) here proposed is presented. This comparison shows the performance and the improvement of estimation results when one takes into account the influence of the bandwidth parameter h which is used by the classical kernel estimators, and when it is discarded by two other nonparametric procedures. Thus, a more general criterion is proposed and it is constructed on the basis of three nonparametric estimators of the residuals distribution, and some new efficient estimators could be built (efficiency in the sense of calculus time)[9], [16] and [17]. Finally, some concluding remarks over the MEE proposed estimators are given in section 6.

2. The general problem of regression

2.1. Linear regression

A variety of applications in signal processing and in instrumentation, are based in the statistical modelling analysis. One of the most used is the linear regression model (simple or multivariable)

$$y_i = x_i^\top \boldsymbol{\theta} + e_i, \quad i = 1, \ldots, n, \tag{1}$$