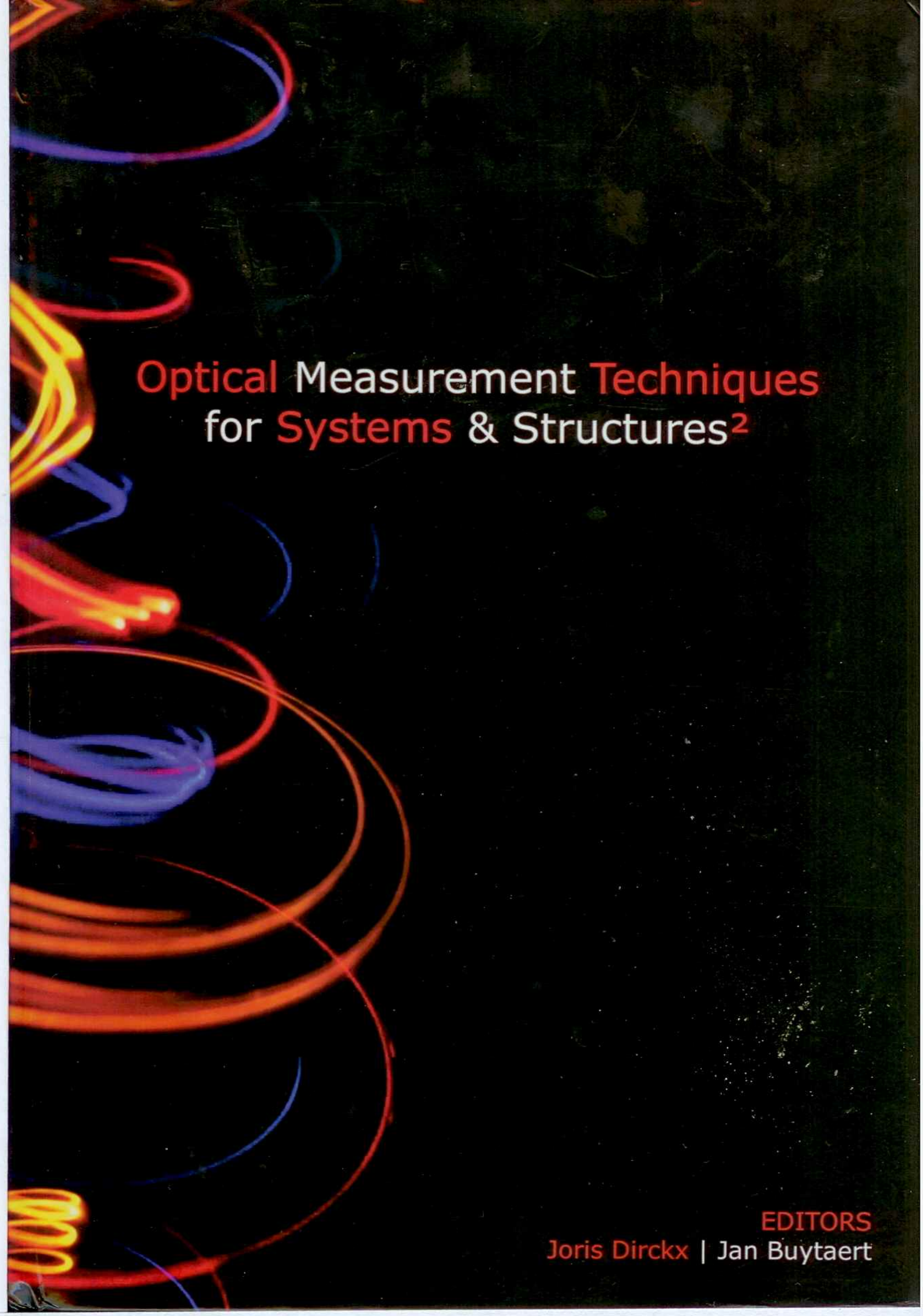

Optical Measurement Techniques for Systems & Structures²

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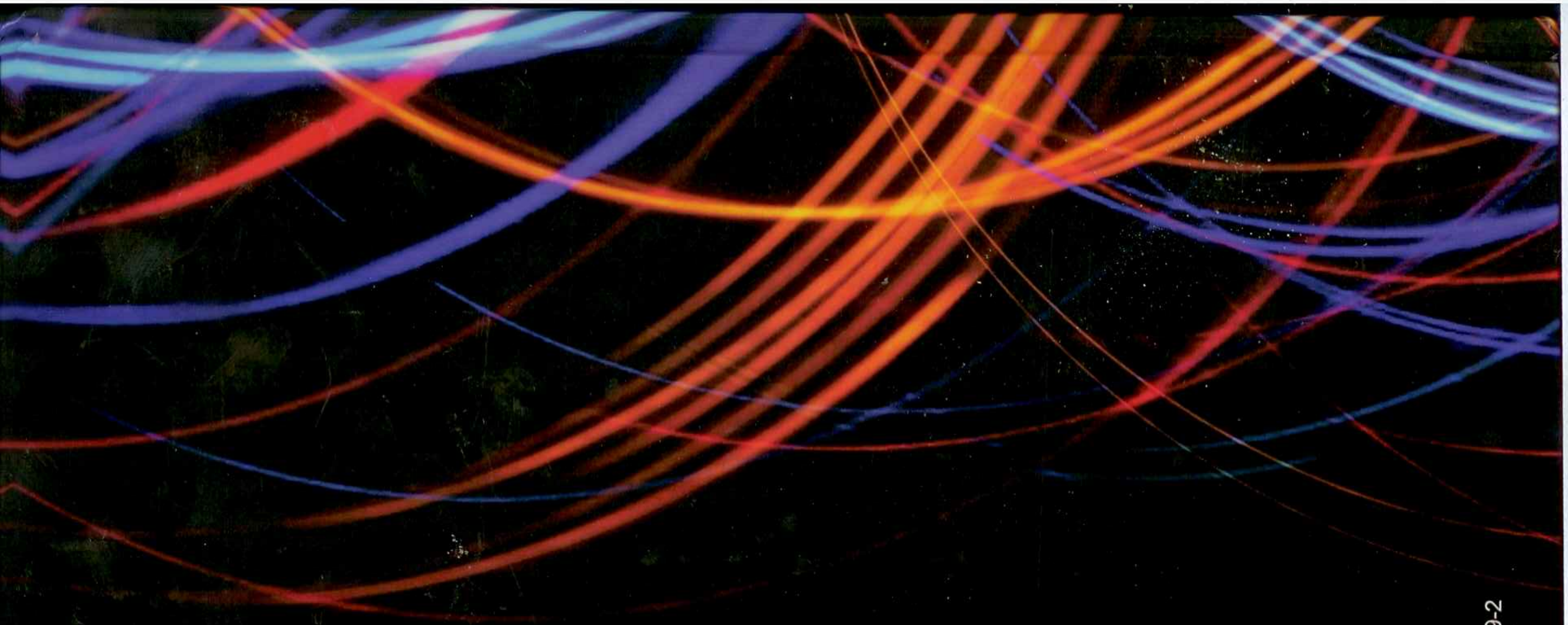

This book gives an overview of the ongoing research and current state-of-the-art of optical metrology. It covers optical measurement techniques and their applications in research, testing and production. The papers in this book use techniques such as interferometry, vibrometry, optical fiber sensors, image correlation techniques, moiré profilometry in application fields such as dimensional metrology, structural mechanics, materials testing and bio-medical applications. This is the second volume of the series.

SHAKER PUBLISHING
ISBN 978-90-423-0419-2

Wavelets Applied to Optical Tomographic Reconstruction

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Abstract

The determination of scalar magnitude distribution associated to refractive index such as temperature and pressure are of interest in many areas of engineering and science. This determination using invasive techniques are slow, does not gives a complete distribution and modifies it when introducing a sensor. We propose a reconstruction method based on optical tomography and wavelets that gives a complete distribution in a faster way than invasive methods.

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Introduction

Optical tomography is a nondestructive and noninvasive technique for obtaining the distribution of refractive index gradients in a cross section of a phase object, PO, in the non-refractive limit from one or more projections. In the case of a axis symmetrical PO, only one projection is necessary [1]. This projection is formed by a set of rays, known as summa rays, which are parallel, as is shown in Figure 1. In the aim of reconstructing a cross-section of a PO from their projections, back projections methods or algebraic reconstruction technique (ART) [2-5] can be used; the proposed method in this paper is a kind of ART.

The application of Optical Tomography to physical quantitative measurements can be divided in four steps:

- Obtaining the interferogram,
- the determination of the phase map from the interferogram,
- the phase inversion to get the refractive index and
- the application of the relationship between the refractive index and the physical magnitude

Many techniques for phase recovery such as Fourier based [7], phase stepping [14] or regularization [15,16,17], provide a non-continuous phase wrapped in the interval $(-\pi,\pi]$. These techniques need an unwrapping processing to recover the right phase. In many cases it is possible to approximate the refractive index using a linear combination of basis functions. This allows, with appropriated weights, to carry out the phase unwrapping and the phase inversion in one step.

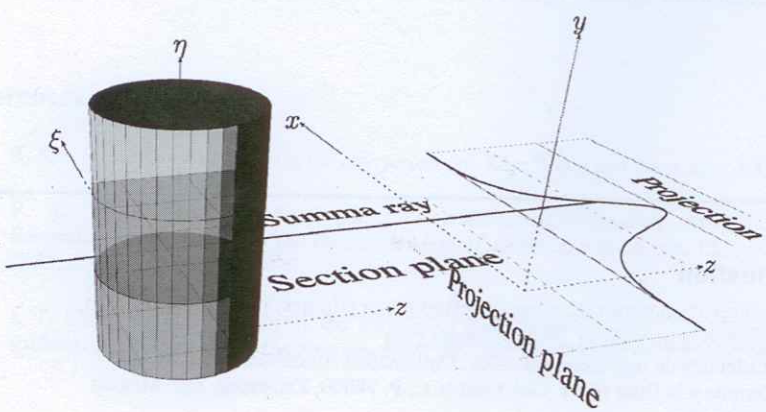


Figure 1: *Graphical description of a projection of a phase object section.*

In algebraic methods the diagram of projections is a linear transformation of the cross section of the object to be reconstructed, i.e. is a matrix equation given by the matrix of projections; the unknown vector, which consists of the image of the cross section and; the solution vector, which corresponds to the projection [2-5].

In the case of selecting Gaussian functions, for optimum approximation, it requires, cf. [13]:

- Positioning the center of the Gaussian functions
- Control the width of each Gaussian function.
- Determine the Gaussian amplitude.

Using wavelets require only the control of the amplitude of each of the basis functions to achieve a perfect reconstruction. The aim of this work is to obtain a fast numeric estimation of refractive smooth index from a wrapped phase, using wavelets as basis functions.

The next section of this paper refers to some concepts of reconstruction in optical tomography. Also some useful definitions in the interpretation of interferograms of axis symmetric PO are introduced. After this, the proposed numerical method is described and compared with a classical method. Also, an example is presents by means of a numerical simulation of a phase object and the technique was applied to a low concentration sodium chloride solution. Finally, conclusions are given in the last section.

Optical tomography

Interferometric techniques are used to measure physical quantities [1,6], such as temperature, pressure or strain. All of them are associated with the distribution of the refractive index; these techniques produce a fringes pattern modulated by variations in the refractive index. The intensity of an interferogram can be represented by the expression

$$I_r = a_r + b_r \cos[2\pi f_0 x + \phi_r] \otimes \rho_r \tag{1}$$

where $r = (x, y)$ are spatial coordinates, a_r is the backlight, b_r is the amplitude modulation and ϕ_r the phase of the wave front associated with the refractive index; f_0 is the frequency of the carrier [7] and ρ_r represents the noise. The symbol $\otimes$ indicates that noise can be additive or multiplicative. In the case of speckle pattern interferometry (SPI) or a single path interferometry, noise is multiplicative [1,5]. When the noise is significant it is necessary to use some filtering methods [18]. In many cases and particularly in this work, both a_r and b_r vary slowly. Then the fringes pattern can be described by

$$I_r = \cos[\phi_r] + \rho_r \tag{2}$$

The phase is wrapped when a method such as Takeda [7] for recovering the phase is applied.

The phase unwrapping problem

Defining φ_r and ϕ_r the wrapped and the unwrapped phase respectively, where $r=(x,y)$ is the vector in a discrete grid, the relationship between these two phases is established by

$$\varphi_r = W\{\phi_r\} = \phi_r + 2\pi k_r \tag{3}$$

where W represents the wrapping operator and k_r represents a field of integers. The phase discrete gradient field, $\nabla\varphi_r$, is defined as

$$\nabla\varphi_r = (\varphi_r - \varphi_s, \varphi_r - \varphi_t) \tag{4}$$

where $s=r-(1,0)$ and $s=r-(0,1)$ are respectively contiguous horizontal and vertical points. In a similar manner the unwrapped discrete gradient field can also be defined as $\nabla\phi_r = (\phi_r - \phi_s, \phi_r - \phi_t)$. For discrete phase fields, closely related to digital fringe images, the problem of the recovery of φ_r and ϕ_r can be properly solved if the sampling theorem is reached, that is, if the distance between two fringes is more than two pixels (the phase difference between two fringes is 2π). In phase terms the sampling theorem is reached if the phase difference between two pixels is less than π. In general

$$\|\nabla\phi_r\| < \pi \tag{5}$$

If this condition is satisfied, the following relation can be established:

$$\nabla\phi_r = W\{\nabla\varphi_r\} = (W\{\varphi_r - \varphi_s\}, W\{\varphi_r - \varphi_t\}) \tag{6}$$

Note that ϕ_r can be obtained from the observed field. By analyzing this equation, it can be seen that ϕ_r can be achieved by two-dimensional integration of the vector $W\{\varphi_r\}$. One way to find out ϕ_r is by means of a least-squares approach [8-10].

Optical tomographic reconstruction

The optical path length δ of a single ray across a transparent medium is represented as

$$\delta = \int_C n\,ds \tag{7}$$

which is the integral of the refractive index (n) along the path of the ray C. When the refraction is not significant, the path of the ray can be seen as a straight line. If the ray propagates along the z axis, as is illustrated in Figure 2, the optical path can be expressed as

$$\delta(\xi,\eta) = \int_C n(\xi,\eta)\,dz \tag{8}$$

and the optical path difference (OPD) $\Delta(\xi,\eta)$ is given as

$$\Delta(\xi,\eta) = \int [n(\xi,\eta) - n_0]\,dz \tag{9}$$

where n_0 is the refractive index of the surrounding medium and $\Delta(\xi,\eta)$ is related to the phase $\phi(x,y)$ in the plain of the projection through the transformation

$$\phi(x,y)=\frac{2\pi}{\lambda}\Delta(\xi,\eta) \qquad (10)$$

In the particular case of a radially symmetrical PO and considering a section of it (illustrated in Figure 2), Equation (7) can be expressed in terms of the Abel transform, $\mathsf{A}\{\cdot\}$ [1], as

$$\Delta(\xi)=\Delta(\xi,\eta=cte)=\mathsf{A}\{n(r)\}=2\int_{\xi}^{+\infty}\frac{[n(r)-n_0]r}{\sqrt{r^2-\xi^2}}dr \qquad (11)$$

where r is the radial coordinate given by $\sqrt{\xi^2+z^2}$.

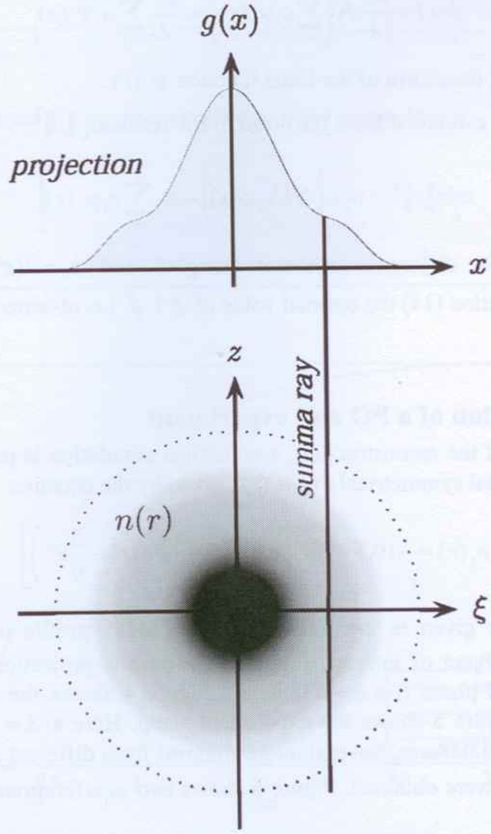


Figure 2: *The Abel Transform applied on a section of a phase object.*

Proposed reconstruction method

As it is well known, in the ART methods, the cross section of the object is superimposed on a logic grid of $M \times N$ elements [3]. Each of these elements or pixels is an unknown [2-5,11,12]. To reconstruct the section, equations should be established to allow the $M \times N$ unknowns of the array being known. These methods are general and not oriented to a specific distribution. For radially symmetrical objects, the generator function is divided into M elements.

The PO $n(r)$ can be approximated by a linear combination of k wavelets basis functions $\psi_i(r)$ i.e.

$$\tilde{n}(r) \approx \sum_{i=1}^{k} a_i \psi_i(r) \quad (12)$$

here r is a radial coordinate, a_i are the wavelets coefficients. From Equations (10) (11) and (12) the phase $\phi(x)$ can be expressed as

$$\phi(x) = \frac{2\pi}{\lambda} \mathrm{A}\left\{ \sum_{i=1}^{k} a_i \psi_i(r) \right\} = \frac{2\pi}{\lambda} \sum_{i=1}^{k} a_i \Psi_i(x) \quad (13)$$

where $\phi(x)$ is the Abel transform of the basis function $\psi_i(r)$.

The phase $\phi(x)$ can be estimated from the norm of the residual, $\|\rho\|^2$

$$\min_{b_i} \|\rho\|^2 = \min_{b_i} \left\| \mathrm{W}\{\Delta_x \varphi(x)\} - \Delta_x \sum_{i=1}^{k} b_i \psi_i(r) \right\|^2 \quad (14)$$

where Δ_x is the finite difference operator along x and $b_i = 2(\pi/\lambda)a_i$ are the wavelets coefficients. From equation (14) the optimal value of ϕ (ϕ^*) is obtained.

Numerical Simulation of a PO and experiment

To assess the quality of the reconstruction, a numerical simulation is performed. A wavefront is projected through a radial symmetrical PO $n_t(r)$, given by the equation

$$n_t(r) = -10.5 \times 10^{-5} \left[\exp\left(-5r^2\right) + \exp\left(-\frac{5}{9}r^2\right) \right] \quad (15)$$

where $r = \sqrt{\xi^2 + z^2}$ is given in cm. This refraction index profile is proposed because it is closely related to an object of interest. Figure 3 shows one projection section of PO and the corresponding wrapped phase (for $\lambda = 632.8nm$). Figure 4 shows the obtained reconstructions using haar wavelet. Figure 5 shows the experiment setup. Here a $\lambda = 632.8nm$ laser was used with a $\Delta\xi = \xi_i - \xi_{i-1} = 0.038mm$. Several interferograms from different concentrations of sodium chloride in water (PO) were obtained. Figure 6 shows two interferograms using two low sodium chloride concentrations.

The interferograms show that the concentration of sodium chloride is homogeneous in the water. Using the proposed method, the refractive index is estimated (shown in Figure 7).

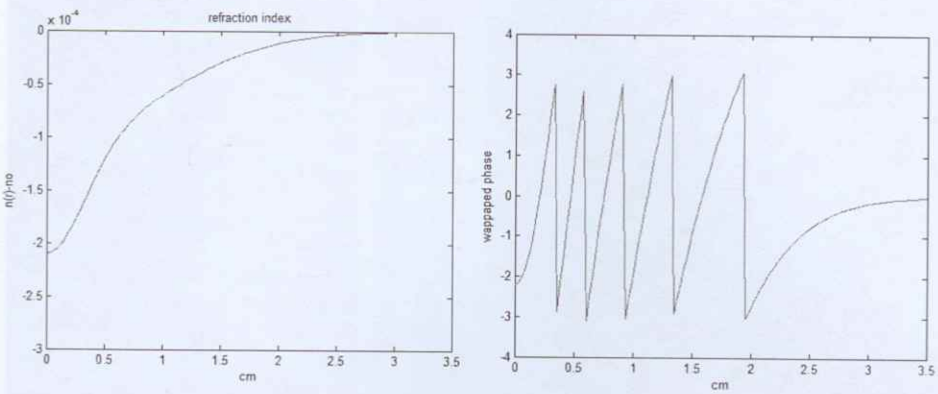


Figure 3: *Phase object test and projected wrapped phase.*

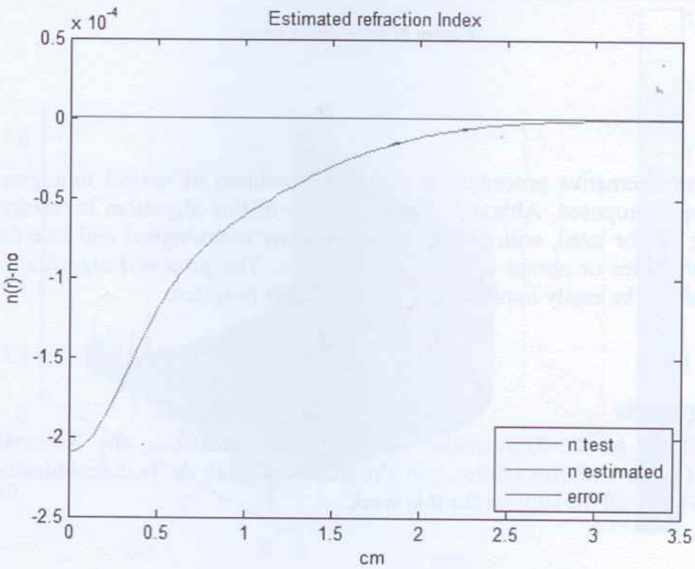


Figure 4: *One dimension synthetic reconstruction.*

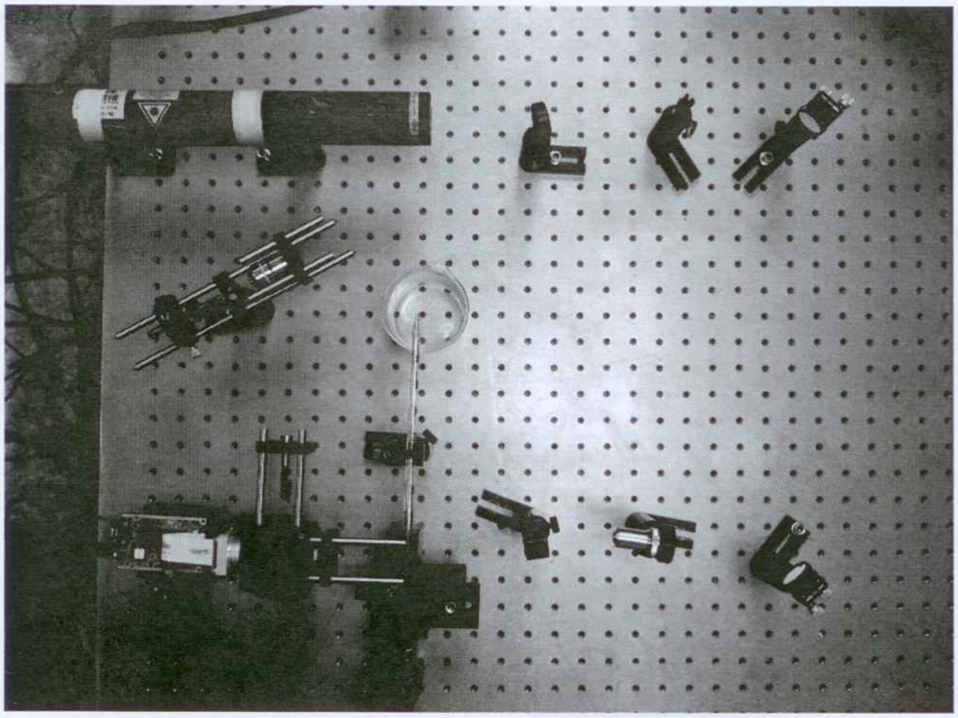


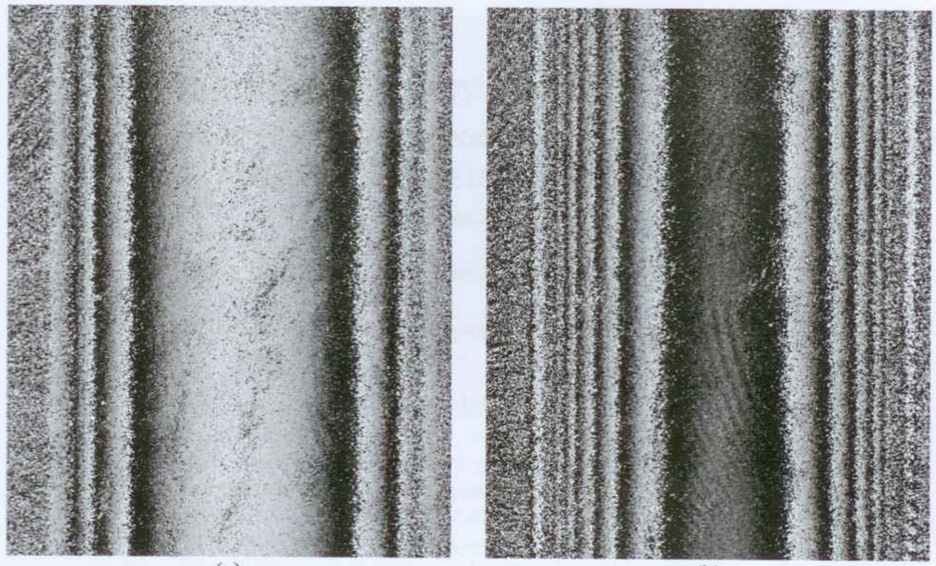
Figure 5: *Experiment setup.*

Conclusion

In this paper an alternative procedure to solve the problem of optical tomographic by using wavelets has been proposed. Although the application of this algorithm is restricted to smooth phase fields, it can be used, with good results, in many metrological and interferometric tests without discontinuities or abrupt changes in the phase. The proposed algorithm is fast, offers good results and can be easily implemented in a computer program.

Acknowledgments

We want to thank to the Universidad Autónoma de Zacatecas, the National Institute of Astrophysics, Optics and Electronics and Programa Integral de Fortalecimiento Institucional 2011 (PIFI 2011) for all the support for this work.



(a) (b)

Figure 6: *Interferograms obtained from two different water sample with very low sodium chloride concentrations. The concentration of sample (a) is lower than the concentration of sample (b).*

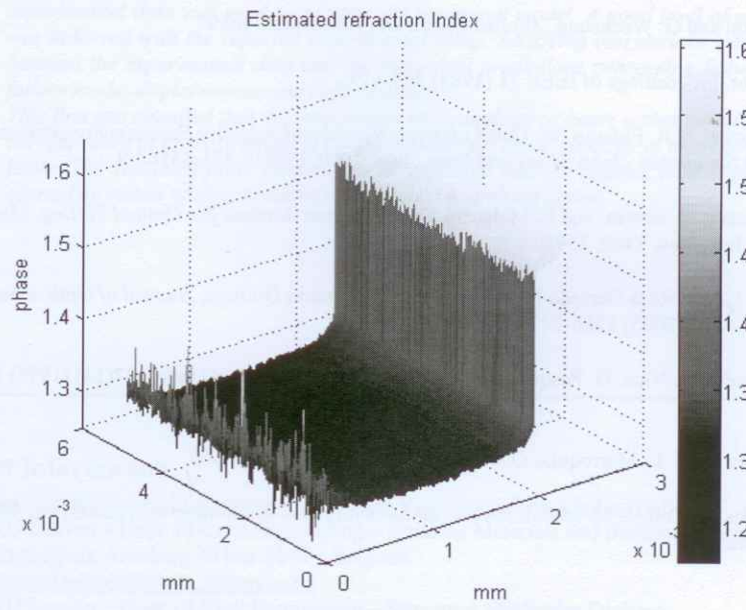


Figure 7: *The solution refractive index average is about 1.33.*

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