Markov Chain Monte Carlo Posterior Density Approximation for a Groove-Dimensioning Purpose

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***Abstract*—The purpose of this paper is to present a new approach for measurand uncertainty characterization. The Markov chain Monte Carlo (MCMC) is applied to measurand probability density function (pdf) estimation, which is considered as an inverse problem. The measurement characterization is driven by the pdf estimation in a nonlinear Gaussian framework with unknown variance and with limited observed data. These techniques are applied to a realistic measurand problem of groove dimensioning using remote field eddy current (RFEC) inspection. The application of resampling methods such as bootstrap and the perfect sampling for convergence diagnostics purposes gives large improvements in the accuracy of the MCMC estimates.**

***Index Terms*—Gibbs sampling, indirect measurement, Markov chain Monte Carlo (MCMC), Metropolis–Hastings (M–H), nonlinear regression, perfect sampling, weighted bootstrap.**

I. Introduction

In many industrial applications, direct access to a measurand (m) is not possible; this is due to the inability to use transducers to measure m directly for any reason such as harsh environment, long distance, etc. Thus, the measurand process must be considered as an inverse problem [1], since the measurand estimation is needed. The characterization of all statistical knowledge upon this quantity of interest is naturally driven by the probability density function (pdf) $\wp(m)$. Probabilistic inferences using Markov chain Monte Carlo (MCMC) methods are considered as another Monte Carlo simulation technique, and other measurand pdf estimation alternatives, by using a fully Bayesian framework [2]–[5]. The "Metropolis algorithm" has been used to solve difficult problems in statistical physics for over 40 years (1953). A generalization of this algorithm is introduced by "Metropolis–Hastings (M–H) algorithms" (1970). In the last few years, other related methods of "Gibbs sampling" have been applied to problems of statistical inference. The MCMC methods, such as Gibbs sampling and M–H algorithms, are powerful Markov-chain methods to simulate multivariate distributions and they have a real impact on Bayesian statistics [6], [7]. The M–H algorithms have been extensively used in physics and, more recently, exploited by statisticians [8].

The importance of Monte Carlo methods for inference problems in signal processing has grown in recent years (*Trans. on Signal Process.,* vol. 50, no. 2, Feb. 2002). This is due to the explosive increase in accessible computing power. Monte Carlo methods could be widely exploited, since one has the necessary computational resources, and these methods can significantly be used in a large class of problems addressed in practice. Monte Carlo methods have also a great degree of flexibility for the solution of challenging computational problems, such as optimization and integration. These kinds of problems abound in statistical signal processing. Literature [9], [10] show the potential usefulness of MCMC in signal processing. For the solution of our measurand problem, we have successfully used hybrid algorithms, firstly, an M–H algorithm for sampling from a complex likelihood density function, and secondly, the Gibbs algorithm for sampling from the posterior density.

An important problem in MCMC is the convergence surveillance of such methods. The convergence performance of MCMC can be improved by using a resampling scheme, e.g., the weighted bootstrap used in [11] (see also [12] and [13], which suggest a class of weighted-bootstrap techniques) and perfect-simulation [14] procedures (see also [15] and [16]). The final interest is to apply the MCMC methods in a realistic problem of indirect measurement (measurand estimation). The remainder of the paper is organized according to the following sections: Section II presents the general formulation of the problem of measurand estimation and the MCMC idea extended to the measurand uncertainty characterization. The Bayesian framework for parameter and measurand estimation is described in a more specified way in Section III, jointly with the classical MCMC uncertainty characterization. The analysis of convergence by resampling and perfect-sampling methods [coupling from the past (CFTP)] are briefly described in Section IV. A measurement complex problem of groove dimensioning using remote field eddy current (RFEC) inspection is given in Section V, and finally, some concluding remarks are given in Section VI.

II. MCMC for Measurand PDF Estimation

The problem of pdf estimation for an indirect measurand is considered in this paper. This problem has been analyzed for a nonlinear Gaussian framework, and the results give the possibility to take up again the problem of pdf estimation in a more suitable or realistic framework (nonlinear Gaussian with unknown variance or non-Gaussian). In many applications, an unknown quantity m has to be estimated from a vector

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