Regularized quadratic cost function for oriented fringe-pattern filtering

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We use the regularization theory in a Bayesian framework to derive a quadratic cost function for denoising fringe patterns. As prior constraints for the regularization problem, we propose a Markov random field model that includes information about the fringe orientation. In our cost function the regularization term imposes constraints to the solution (i.e., the filtered image) to be smooth only along the fringe's tangent direction. In this way as the fringe information and noise are conveniently separated in the frequency space, our technique avoids blurring the fringes. The attractiveness of the proposed filtering method is that the minimization of the cost function can be easily implemented using iterative methods. To show the performance of the proposed technique we present some results obtained by processing simulated and real fringe patterns. © 2009 Optical Society of America

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The demodulation of digital fringe patterns is widely used in optical tests such as electronic speckle pattern interferometry (ESPI), holographic interferometry, or moiré interferometry. Several techniques can be applied for the extraction of the phase field; however, in the process of formation and acquisition of fringe patterns, noise commonly contaminates images. For this reason denoising fringe patterns plays an important role to make phase extraction easier, more robust, and more accurate. However, the frequencies of fringes and noise usually overlap and normally cannot be separated properly, and common filters for image processing have blurring effects on fringe features, especially for patterns with high-density fringes. For these cases the use of anisotropic filters is a better way for removing noise without blurring effects.

In the field of image processing, the regularization theory [1–3] has been demonstrated to be a powerful tool for reconstructing images. Particularly, in the past few years some works have been developed for fringe analysis, among them are the works in [4,5]. Although directional filtering has been studied for fringe images, for example, the outstanding work by Tang *et al.* [6] that proposed second-order oriented partial-differential equations for denoising ESPI fringes, we use a different and powerful mathematical tool for this purpose. In this Letter we derive a regularized quadratic cost function that is used for denoising along fringes in this kind of images.

It is widely known that the problem of reconstructing an image x from a degraded image y, i.e., the observed image, is often formulated according to the model

$$y = H(x) + n, \tag{1}$$

where n is the additive noise and H may represent a linear operator that is assumed to be known, which may be some kind of distortion. For example, H may be the point spread function of the imaging system. In general, the information provided by the observations of y is not enough for a proper estimation of x. For an adequate recovery of x we need to regularize the problem including prior information about the characteristics of the field to be estimated. The stochastic route to regularize the problem described in Eq. (1) may be derived using the Bayesian estimation. Using the Bayes's rule, one may model the posterior distribution of x with a given y as

$$P_{x/y}(x) = KP_{y/x}(x)P_x(x), \tag{2}$$

where K is a constant and

$$P_{y/x}(x) = K_1 \exp\left[-\sum_{m\in L} \Phi(H(x_m) - y_m)\right] \tag{3}$$

represents the conditional distribution of y with a given x, Φ is a potential function that is defined by the noise model, $m=(i,j)$ is the image coordinates in a regular lattice L, and K_1 is a constant. The prior distribution $P_x(x)$ that is commonly used in the framework of the Bayesian regularization are the Markov random fields (MRFs) [3,7,8], which are defined by a set C of potential functions V_c that ranges over the cliques associated with a given neighborhood system. An important characteristic of the MRFs is that the probabilistic dependencies of the elements of the estimated field are local, which make MRFs adequate for modeling piecewise smooth functions. Using an MRF, $P_x(x)$ is then given by the Gibbs distribution

$$P_x(x) = K_2 \exp\left[-\sum_{C} V_c(x_m)\right], \tag{4}$$

where K_2 is a normalizing constant.

Then, we can define the maximum *a posteriori* es-

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