ACCELERATED EXPANSION OF THE UNIVERSE VIA ENERGY CONSERVATION

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This paper analyzes the boundary term of the Hilbert-Einstein action for an FRW metric, and uses it to set the conditions of the Einstein's Field Equations. By means of the energy conservation equation, we identify a new non gravitational general relativistic effect that modifies the space-time fabric. This effect is due to the energy density that surrounds a given space-time region, and it gives a physical explanation to the accelerated expansion of the universe. It also explains why we have not found any particle or fluid responsible of the dark energy and it clarifies the coincidence problem. These explanations are achieved without assuming the existence of exotic matter of unphysical meaning or having to modify the Einstein's Field Equations.

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The Einstein Field Equations (EFE), describe the fundamental interactions of gravitation as a result of space-time being curved by means of the mass and energy inside this space-time. In this sense the metric and the stress-energy tensor determine the system, and it can be obtained via the variation principle of the Einstein Hilbert (EH) action plus the matter action field.

Since the discovery of Edwin Hubble, in 1929, which established that the light from farther galaxies was red-shifted, inferring expansion of the universe; scientists have been proposing many models and theories, in order to explain the expansion phenomenon. Basically, such explanations can be sorted into two different kinds [1]: one is that the problem lies on the matter content, i.e. the stress-energy tensor, $T_{\mu\nu}$, and that there must be some extra energy fluid that may fill the space-time, e.g., quintessence [2]; another possibility states that the problem lies on the geometric sector, implying that Einstein's theory could be wrong or incomplete e.g., Modified Gravity [3].

In this paper assume that the structure of the Einstein's Field Equations is correct, nevertheless, by analyzing the boundary conditions on the variation of the Einstein Hilbert action, we identify a problem on the matter content introduced to the system. Taking into account the energy density outside certain space-time region, a new relativistic effect, never considered before, is derived. The proposed model has the desired futures of the ΛCDM model, explaining the accelerated expansion of the universe [4–6], without adding extra exotic matter.

The variation of the Einstein Hilbert action plus the matter field is derived, in order to analyze the boundary conditions on the surface term for an FRW metric.

The action is given by,

$$S = \int [\frac{1}{2\kappa}R + \mathcal{L}_M]\sqrt{-g}d^4x, \tag{1}$$

for an extremum, the variation yields,

$$\begin{aligned} 0 &= \delta S \\ &= \int \frac{1}{2\kappa}[(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R)\delta g^{\mu\nu}\sqrt{-g}d^4x \\ &\quad + \int \frac{1}{2\kappa}(g^{\mu\nu}\delta R_{\mu\nu})\sqrt{-g}d^4x + \int \delta(\sqrt{-g}\mathcal{L}_M)]d^4x. \end{aligned} \tag{2}$$

In order to get the EFE, we must eliminate the middle term,

$$\int_V \nabla_\alpha A^\alpha \sqrt{-g}d^4x, \tag{3}$$

which is an integral of a divergence, $\nabla_\alpha A^\alpha$, over the volume, $\sqrt{-g}d^4x$, that can be converted to a surface integral by the Stokes's theorem.

The contribution of the integral depends on the boundary conditions [7].

$$\int_V \nabla_\alpha A^\alpha \sqrt{-g}d^4x = \int_{\partial V} A^\alpha n_\alpha \sqrt{|h|}d^3x \tag{4}$$

where $g^{\mu\nu}\delta\Gamma^\alpha_{\mu\nu} - g^{\mu\alpha}\delta\Gamma^\nu_{\mu\nu} \equiv A^\alpha$ is a rank 1 tensor, n_α is the vector perpendicular to the surface and h is the induced metric in $3-D$.

There are several ways to ditch the term: if the underlying space-time manifold has a boundary, a counter-term [8] [9] is added to the action (1). The usual approach for a closed space-time manifold is to set this boundary term to be zero, because the variations are assumed to vanish on the surface of V [10].

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For the flat FRW metric, the derivation is

$$\begin{aligned} A^\alpha n_\alpha &= \left(g^{\mu\nu}\delta\Gamma^\alpha_{\mu\nu} - g^{\mu\alpha}\delta\Gamma^\nu_{\mu\nu}\right)n_\alpha \\ &= \left((g^{rr}\delta\Gamma^t_{rr} + g^{\phi\phi}\delta\Gamma^t_{\phi\phi} + g^{\theta\theta}\delta\Gamma^t_{rr}) \right. \\ &\left. - (g^{tt}\delta\Gamma^r_{tr} + g^{tt}\delta\Gamma^\phi_{t\phi} + g^{tt}\delta\Gamma^\theta_{t\theta})\right)n_t. \end{aligned} \tag{5}$$

The contribution of the term (4) evaluated at the border,

$$\int_{\partial V} A^\alpha n_\alpha \sqrt{|h|} d^3x = 6 \int_{\partial V} \delta \left(\frac{\dot{a}(t_f)}{a(t_f)}\right) \sqrt{|h|} d^3x, \tag{6}$$

is zero when the variation of $\dot{a}(t_f)/a(t_f)$ vanishes. For purpose of identification we will rename the boundary term $a(t_f) = b$.

The problem lies on the source term. Usually, the only energy density content included is the one that is inside the volume bounded by the hypersurface cast by the radius $a(t)$ This is done under the assumption that the matter content outside this radius does not affect gravitationally the space-time. The real size of the system is given by the conditions of Eq (6). The lack of inclusion of the energy density surrounding the given volume leads us to incomplete conservation equations; consequently, incomplete Friedmann equations.

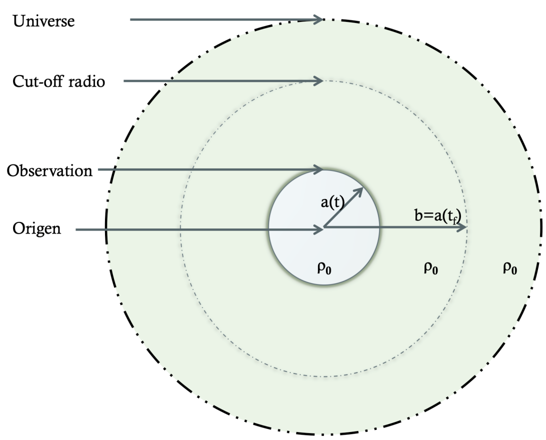

FIG. 1. Representation of a spherical system bounded by a radio b, with total volume $V_T = \frac{4}{3}\pi b^3$ and total mass M_T, with two same density sub-regions: one spherical concentric region of volume $V = \frac{4}{3}\pi a(t)^3$ and mass m, and another concentric shell of volume $V' = \frac{4}{3}\pi(b^3 - a(t)^3)$ and mass m'.

The Friedmann equation for a homogeneous and isotropic flat space-time is,

$$3\left(\frac{\dot{a}}{a}\right)^2 = 8\pi G(\rho_T), \tag{7}$$

where ρ_T is the total density of the complete system. Being the space-time homogeneous at large scales, this density, is the same for the whole space-time, and it satisfies the conservation equations. The total energy of the system is, $U = M = m+m'$, where m is the interior mass, and m' the exterior mass, as seen on Fig. 1; therefore the fluid of the whole closed system satisfies

$$d(\rho V + \rho' V') = -p dV_T. \tag{8}$$

By aid of the first law of thermodynamics for an adiabatic closed system, we get the evolution of a perfect fluid on the inner subsystem

$$d(\rho V) = -p dV. \tag{9}$$

So the fluid equation for the inner subsystem is

$$\dot{\rho} = -3\frac{\dot{a}}{a}(\rho + p), \tag{10}$$

with aid of equation of state for a perfect fluid, $\omega\rho = p$, we arrive to the known solution

$$\rho_\omega = \rho_{0\omega} a^{-3(1+\omega)}. \tag{11}$$

The exterior subsystem is also a closed one, so the thermodynamics law for it is given by:

$$d(\rho' V') = -p' dV'. \tag{12}$$

If we add up the equations of both sub systems (9) and (12), we obtain the conservation of the whole system (8).

With the aid of the equation of state for a perfect fluid, we get the equation of the evolution of the density for the exterior region

$$\dot{\rho}' = -3\frac{b^2\dot{b} - a^2\dot{a}}{b^3 - a^3}\rho'(1+\omega), \tag{13}$$

whit aid of the boundary conditions, $\dot{b}/b = 0$, we arrive to,

$$\rho'_\omega = \rho_{0\omega}\left(\frac{b^3}{b^3 - a^3}\right)^{(1+w)}. \tag{14}$$

This is how the energy density outside a given radius, $a(t)$, modifies the space-time. For a large outer radius, b, this parameter becomes constant,

$$\rho'_\omega = \rho_0. \tag{15}$$

Being all the matter content analyzed, we proceed to contrast the effects on the dynamics of the universe. First we present a general solution, then, a particular solution for a large outer radius.

Introducing the results of conservation equations for the inner (11) and outer (14) regions, the Friedmann equation (7) is:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_{0\omega} a^{-3(1+\omega)} + \frac{8\pi G}{3}\rho_0\left(\frac{b^3}{b^3 - a^3}\right)^{(1+w)}, \tag{16}$$

the LHS of the equations gives the kinetic information, the first RHS equation gives the information of the gravitational potential, while the last term acts as an energy reservoir or a spring potential.

The acceleration equation results,

$$\begin{aligned}\frac{\ddot{a}}{a} &= -\frac{4\pi G}{3}\rho_{0\omega}a^{-3(1+\omega)}(1+3w) \\ &+ 4\pi G\left(\frac{a^3}{b^3-a^3}\rho_0(1+\omega)+\frac{2}{3}\rho_0\left(\frac{b^3}{b^3-a^3}\right)^{(1+w)}\right),\end{aligned} \tag{17}$$

where the first term on the RHS of equations (16) and (17) gather all the different fluids inside the inner region, including, possibly, the dark matter; and the last term on both equations, gather all the different kinds of matter in the outer region; the matter contents depend on the value w of the equation of state.

It is worth noting the positive sign of the last term on the acceleration equation (17), which is responsible for the accelerated expansion of the universe, due to the energy density outside of it.

If a large outer radius is assumed, $b \gg a(t)$ for any given time, the Friedmann equations become:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_\omega + \frac{8\pi G}{3}\rho_0, \tag{18}$$

and

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho_\omega(1+3p) + \frac{8\pi G}{3}\rho_0. \tag{19}$$

We could consider the outer region to act like an energy reservoir; under these circumstances, the outer energy density acts much like the cosmological constant. This is certainly not a gravitational effect, it is a general relativistic effect that gives the information on how the outer mass-energy density stretches out the space-time fabric, which explains why there is not such a dark energy fluid or particle.

These equations resemble the Friedman Equations of a ΛCDM model, with the advantage of one less parameter to be fixed in the model, since ρ_0 is obtained by the system conditions. The energy density term, ρ_0, is of the same order of magnitude as the ordinary matter content of the universe, ensuring homogeneity, explaining the Cosmological Coincidence Problem [11].

From the Sloan Digital Sky Survey and the Planck satellite, we estimate that the whole universe is at least 250 times the radius of the observable one, which is more than enough in this model to ensure isotropy on the latter.

Instead of a dark fluid, we visualize the universe in a thermal energy reservoir, represented by fluid with density ρ_0 in a vast outer region that stretches out evenly the space-time of the inner region.

[1] A. Joyce, L. Lombriser, and F. Schmidt, Annual Review of Nuclear and Particle Science **66**, 95 (2016).
[2] B. Ratra and P. J. E. Peebles, Phys. Rev. D **37**, 3406 (1988).
[3] T. Clifton, P. G. Ferreira, A. Padilla, and C. Skordis, Physics Reports **513**, 1 (2012), modified Gravity and Cosmology.
[4] S. Perlmutter, G. Aldering, G. Goldhaber, R. Knop, P. Nugent, P. Castro, S. Deustua, S. Fabbro, A. Goobar, D. Groom, *et al.*, The Astrophysical Journal **517**, 565 (1999).
[5] A. G. Riess, A. V. Filippenko, P. Challis, A. Clocchiatti, A. Diercks, P. M. Garnavich, R. L. Gilliland, C. J. Hogan, S. Jha, R. P. Kirshner, *et al.*, The Astronomical Journal **116**, 1009 (1998).
[6] P. A. R. Ade *et al.*, A&A **594**, A13 (2016).
[7] R. M. Wald, *General relativity* (Chicago Univ. Press, Chicago, IL, 1984).
[8] J. W. York, Phys. Rev. Lett. **28**, 1082 (1972).
[9] G. W. Gibbons and S. W. Hawking, Phys. Rev. D **15**, 2752 (1977).
[10] S. M. Carroll, *Spacetime and geometry: An introduction to general relativity* (2004).
[11] S. Weinberg, Rev. Mod. Phys. **61**, 1 (1989).