Foundations of Physics Letters, Vol. 11, No. 6, 1998

IS THE FREE ELECTROMAGNETIC FIELD A CONSEQUENCE OF MAXWELL'S EQUATIONS OR A POSTULATE?

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Received February 20, 1998; revised September 26, 1998

It is generally accepted that solutions of so called "free" Maxwell equations for $\varrho = 0$ (null charge density at *every* point of the *whole* space) describe a free electromagnetic field for which flux lines *neither* begin *nor* end in a charge). In order to avoid ambiguities and unacceptable approximation which have place in the conventional approach in respect to the free field concept, we explicitly consider three possible types of space regions: (i) "isolated charge-free" region, where a resultant electric field with the flux lines which *either* begin *or* end in a charge *is zero* in every point, for example, inside a hollow conductor of any shape or in a free-charge universe; (ii) "non-isolated charge-free" region, where this electric [see (i)] field *is not zero* in every point; and (iii) "charge-neutral" region, where point charges exist but their algebraic sum is zero. According to these definitions a strict mathematical interpretation of Maxwell's equations gives following conclusions: (1) In "isolated charge-free" regions electric *free* field cannot be unconditionally understood neither as *a direct consequence* of Maxwell's equations nor as a valid approximation: it may be introduced only as *a postulate*; nevertheless, this case is compatible is the existence of a *time-independent* background magnetic field. (2) In both "charge-neutral" and "non-isolated charge-free" regions, where the condition $\varrho = \delta$ function or $\varrho = 0$ respectively holds, Maxwell's equation for the total electric field have non-zero solutions, as in the conventional approach.

However, these solution cannot be strictly identified with the *electric* free field. This analysis gives rise to the reconsideration of the free-electromagnetic field concept and leads to the simplest implications in respect to charge-neutral universe.

Key words: free field, empty space, charge-free region, massive photon.

1. INTRODUCTION

It is well-known that the set of four Maxwell's equations (ME) [1,2] describes different phenomena according to particular initial and boundary conditions (BC). The authors of this note have independently found that the structure of solutions of ME may be different that it is conventionally believed [3-5]. As part of the process to establish BC for our generic problem, we explore here the meaning of the solutions of ME in regions of space with null charge density ($\varrho = 0$).

Conventionally, $\varrho = 0$ in *every* point of the *whole* space represents "empty space" (see, e.g., [1], page 331, or [2], §46). Under this condition, both Eqs. (5) and (6) (see below) describe solenoidal fields, which imply that the electric and magnetic fields (**E** and **H**) in that region of space are transverse to the *instantaneous* [6] direction of propagation. Moreover, since there are no charges in such region, the electromagnetic wave corresponds to a so called *free* field, whose flux lines neither begin nor end in a charge.

Note that there is a uncertainty[1] in the definition of so called *free* electric field in text-books and monographs. For example, from the one hand, one can find in [2] (§46) that non-zero solutions of so called *free Maxwell equations* admit us to claim that an moving-charge-independent electric field can exist. From the other hand, in [7] (§97) it is claimed that displacement currents $\left(\frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}\right)$ cannot exist independently on a movement of charges. In turn in [2] (§62) it is also proved that a field, radiated by a system of a moving charges, depends on these charges (retarded potentials).

Let us elucidate this situation. We consider inhomogeneous wave equations in the potential form:

$$\Delta \mathbf{A} - \frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2} = -\frac{4\pi}{c}\mathbf{j}, \tag{1}$$

$$\Delta \varphi - \frac{1}{c^2}\frac{\partial^2 \varphi}{\partial t^2} = -4\pi\varrho. \tag{2}$$

[1] It would be better to say "a confusion"!

Now let us consider general solutions of these equations [2][2]

$$\varphi = \int \frac{\varrho_{(t-R/c)}}{R} dV + \varphi^*, \tag{3}$$

$$\mathbf{A} = \frac{1}{c} \int \frac{\mathbf{j}_{(t-R/c)}}{R} dV + \mathbf{A}^*, \tag{4}$$

where φ^* and $\mathbf{A}^*$ are general solutions of (1), (2) without a *rhs*.

These solutions (without φ^* and $\mathbf{A}^*$) represent the field produced by the system, while φ^* and $\mathbf{A}^*$ must be set equal to the *external* field acting on the system. Note that in *any* text-books[3] and monographs fields φ^* and $\mathbf{A}^*$ are identified with a radiation *falling* on a system. The system under consideration consists of moving charges and fields, the latter produced by moving charges inside an arbitrary and fixed volume V. In other words, field components φ^* and $\mathbf{A}^*$ do not depend on currents $\mathbf{j}$, i.e., this field cannot be associated with the moving charges. In the terminology accepted in the conventional approach, it is so called *free field*. Then the question arises: *where does the field* $\{\}^*$ *come from?* In finding an answer one may suggest that this field is produced by currents which are placed outside our system. Nevertheless, it is not the only suggestion because nobody will forbid us to insert these currents into our system. Thus, using once again Eqs. (3) and (4), we obtain *another* solution $\{\}^*$ which *in no way* depends on currents of our *new* system as it was assumed in the previous case! We can continue this reasoning infinitely (i.e. $V \to \infty$). Then, after having extended the integration over *all* space, there will be no place left for external sources used in the conventional approach for justifying a free field concept. In other words, *where does this free field* $\{\}^*$ *come from?* It may mean that free field either does not exist or exists *always* and it cannot be produced by any current!

We want to argue here that such "free-field" interpretation is not completely consistent with the physics behind ME. The remainder of this note is organized as follows: in Sec. 2 we critically revisit the conventional interpretation to find that $\varrho = 0$ does not lead to obligatory existence the *free electric* field. Sec. 3 explores some implications of our findings and Sec. 4 closes the paper.

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[2] Equations (62.9) and (62.10).

[3] See, for example, the text in fin of §62 [2].

2. THE CONVENTIONAL INTERPRETATION CRITICALLY REVISITED

In CGS units, Maxwell's equations are[4]

$$\nabla \cdot \mathbf{E} = 4\pi\varrho, \tag{5}$$

$$\nabla \cdot \mathbf{H} = 0, \tag{6}$$

$$\nabla \times \mathbf{H} = \frac{4\pi}{c}\mathbf{j} + \frac{1}{c}\frac{d\mathbf{E}}{dt}, \tag{7}$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{d\mathbf{H}}{dt}. \tag{8}$$

Charge conservation is assured by the standard continuity condition:

$$\nabla \cdot \mathbf{j} + \frac{d\varrho}{dt} = 0. \tag{9}$$

We consider three types of regions: (i) "isolated charge-free" region, where a resultant electric field with the flux lines which *either* begin *or* end in a charge *is zero* in every point, for example, inside a hollow conductor of any shape or in a free-charge Universe; (ii) "non-isolated charge-free" region, where this electric [see (i)] field *is not zero* in every point; and (iii) "charge-neutral" region, where point charges exist but their algebraic sum is zero. Usually, one set[5] $\varrho = 0$ in (5) and (7) at the *whole* space (or in "isolated charge-free" region, see (i)) and obtains equations for *free* field. We argue here that this straightforward procedure does not rigorously lead to free-field solution of ME. For our reasoning, it is important to recall how Eqs. (5) and (7) are obtained in the conventional approach.

Let us introduce vectors $\mathbf{E}_0$ and $\mathbf{E}^*$. The vector $\mathbf{E}_0$ represents an electric field with the flux lines which *either* begin *or* end in a charge; the vector $\mathbf{E}^*$ represents a certain free field for which flux lines *neither* begin *nor* end in a charge. According to the Gauss' law [1]: *The flux of the electric field* $\mathbf{E}_0$ *through any closed surface, that is, the integral* $\oint \mathbf{E}_0 \cdot d\mathbf{a}$ *over the surface, equals* 4π *times the total charge enclosed by the surface*:

$$\oint_S \mathbf{E}_0 \cdot d\mathbf{a} = 4\pi Q = 4\pi \sum_i q_i = 4\pi \int_V \varrho\, dv. \tag{10}$$

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[4] As argued elsewhere [4], Eqs. (7)-(9) are expressed as total time derivatives. However, such modification is not important for our argument here, so that they can be substituted by the conventional partial time derivatives. Recall also that $\mathbf{E} = \mathbf{D}$ and $\mathbf{H} = \mathbf{B}$ in vacuum in cgs units.

[5] In (7) $\varrho\mathbf{V} = \mathbf{j}_{cond}$

This statement is *equivalent* to the Coulomb's law and it could be accepted equally well as the basic law of electrostatic interactions, *after* charge and field have been *defined*. In other words, Gauss' and Coulomb's laws are not independent physical laws, but the same law expressed in different ways. Note this well known fact that a proof of Eq. (10) *hinged* on the *inverse-square nature* of the interaction and, therefore, Gauss' theorem (law) in physics makes sense *solely* to inverse-square fields.[6] In this respect, we would like to stress two aspects:

(a) Coulomb's law is defined in terms of the individual q_i, so that the expression for charge Q (Eq.(10)) in terms of charge density ϱ is only strictly valid as a limit when a very large number of charges is present. (We can add that, of course, ϱ may be treated as δ-function).

(b) The right-hand-sides of Eq.(10) may be zero in two different ways: (*) *Charge-free condition*, $Q = 0$ when $q_i = 0$, all "i". (**) *Charge-neutral condition*, $Q = 0$ when $q_i \neq 0$, all "i" independently.

From the mathematical point of view, for cases (*) and (**), one should not expect the same solution for electric field value $\mathbf{E}_0$ on the basis of Gauss' law (10). Indeed, for an *isolated* charge-free region the *only* solution is

$$\mathbf{E}_0 = 0, \tag{11}$$

which simply means that a non-existing charge cannot produce the electric field $\mathbf{E}_0$. Note that previous assertion is *qualitatively* different to saying that there exists an electric field in the region that becomes zero when $Q = 0$.

Let us recall now the formulation of Ostrogradsky-Gauss' theorem. Being valid for every vector field, it certainly holds for $\mathbf{E}_0$:

$$\oint_S \mathbf{E}_0 \cdot d\mathbf{a} = \int_{\mathcal{V}} \nabla \cdot \mathbf{E}_0 \, dv. \tag{12}$$

Both Eq. (10) and Eq. (12) hold for any volume we are allowed to choose—of any shape, size, or location. Comparing them, we see that this can only be true if at *every* point,

$$\nabla \cdot \mathbf{E}_0 = 4\pi\varrho. \tag{13}$$

In the *isolated* charge-free region ϱ is equal to zero by definition. Thus, $\nabla \cdot \mathbf{E}_0$ is automatically zero in every space point of this region because of $\mathbf{E}_0$ is zero in such region.

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[6] Or superposition of such fields.

Now let us recall the origin of the displacement current term in the Eq. (7). Really, Maxwell discovered his famous paradox: without this term Eq. (7) is not compatible with the continuity Eq. (9)):

$$\nabla \times \mathbf{H} = \frac{4\pi}{c}\mathbf{j}_{cond} + (?), \tag{14}$$

so the term (?) is $\mathbf{j}_{disp}$ and it has to satisfy:

$$\nabla \times \mathbf{H} = \frac{4\pi}{c}\mathbf{j}_{tot} = \frac{4\pi}{c}(\mathbf{j}_{cond} + \mathbf{j}_{disp}), \tag{15}$$

$$\nabla \cdot \mathbf{j}_{tot} = \nabla \cdot \mathbf{j}_{cond} + \nabla \cdot \mathbf{j}_{disp} = 0, \tag{16}$$

$$\nabla \cdot \mathbf{j}_{disp} = -\nabla \cdot \mathbf{j}_{cond} = \frac{d\varrho}{dt}. \tag{17}$$

Using (13), one obtains

$$\nabla \cdot \mathbf{j}_{disp} = \frac{1}{4\pi}\frac{d}{dt}\nabla \cdot \mathbf{E}_0 = \nabla \cdot \left(\frac{1}{4\pi}\frac{d\mathbf{E}_0}{dt}\right). \tag{18}$$

The general solution of this equation is

$$\mathbf{j}_{disp} = \frac{1}{4\pi}\frac{d\mathbf{E}}{dt} + \nabla \times \{\mathbf{F}_1(x,y,z,t)\} + \mathbf{F}_2(t) + \text{const}, \tag{19}$$

where $\mathbf{F}_{1,2}$ are arbitrary vectors.

In the conventional approach all additional time-dependent terms but not time derivative are set to zero without any special consideration:

$$\nabla \times \{\mathbf{F}_1(x,y,z,t)\} + \mathbf{F}_2(t) + \mathbf{const} = 0. \tag{20}$$

This is the simplest way to obtain the Eq. (7). Then *after having obtained* the Eq. (7), a next step (*attention!*) is habitually done to establish Maxwell's equations for free field (see, e.g., [2], §46):

In empty space, all terms with ϱ and $\mathbf{j}_{cond} = \varrho\mathbf{V}$ are set to zero at whole space, and Maxwell's equations take the following form

$$\nabla \cdot \mathbf{E}_0 = 0, \tag{21}$$

$$\nabla \cdot \mathbf{H} = 0, \tag{22}$$

$$\nabla \times \mathbf{H} = +\frac{1}{c}\frac{d\mathbf{E}_0}{dt}, \tag{23}$$

$$\nabla \times \mathbf{E}_0 = -\frac{1}{c}\frac{d\mathbf{H}}{dt}. \tag{24}$$

However, as we have already seen, setting $\varrho = 0$ in *the whole space* is equivalent to the imposing *charge-free condition* (when $q_i = 0$ for all i). Strictly speaking, it corresponds only to the *isolated* charge-free region with no field at all:

$$\nabla \cdot \mathbf{E}_0 = 0, \tag{25}$$

$$\nabla \cdot \mathbf{H} = 0, \tag{26}$$

$$\nabla \times \mathbf{H} = (?), \tag{27}$$

$$\nabla \times \mathbf{E}_0 = 0. \tag{28}$$

In other words, the value of electric field $\mathbf{E}_0$ should be zero in every point of this region[7]. Nevertheless, we have to stress here that for non-zero field values (the flux lines of this field *begin or end* in charges) Eqs. (21)-(24) make sense in the case of *non-isolated* charge-free region as well as in the case of charge-neutral region[8]

Let us now, however, do very important remark:

Regardless the formulation of boundary-value problem for Max-well's equations, it is obviously that Gauss' law (10) is invariant with respect to *any* additional vector field $\mathbf{F}^*_{add}(x, y, z, t)$ for which flux lines *neither* begin *nor* end in a charge (for this vector $\nabla \cdot \mathbf{F}^*_{add} = 0$ in *every* point of *whole* space by definition).

In the conventional approach, this zero-divergence term becomes identified with a free *electric* field $\mathbf{E}^*$ in the approximation when charges and currents are found itself very far from the region under consideration. According to this conventional procedure free field has not been derived from basic equations but introduced as an arbitrary term which satisfies Gauss' law (10) or Eq. (13). In the other words, we only can *postulate* an existence of the free field $\mathbf{E}^*$. In this case instead of Eqs. (21)-(24) one, repeating the calculations (14)-(19), obtains *another* displacement current $+\frac{1}{4\pi}\frac{\partial \mathbf{E}^*}{\partial t}$.

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[7] The sense of (?) in Eq. (27) we explain in Sec. 3.1 of Sec. 3.

[8] Note that thus one resolves Maxwell's paradox without introducing *any* field nonconnected with a charge, i.e., without introducing "free field"! Actually, in this case the Maxwell equation (7) one writes as (for a single moving particle [4]):

$$\nabla \times \mathbf{H} = \frac{4\pi}{c} q\delta(\mathbf{r} - \mathbf{r}_q(t))\mathbf{V} - \frac{1}{4\pi}(\mathbf{V} \cdot \nabla)\mathbf{E}_0.$$

(Obviously, $\delta(\mathbf{r} - \mathbf{r}_q(t))$ in "non-isolated charge-free" region is zero). So, Maxwell paradox is resolved but it is obviously that "free" electric field $\mathbf{E}^*$ cannot be solution of *this* equation by definition.

Then, setting $\varrho = 0$ in the whole space, one obtains

$$\nabla \cdot \mathbf{E}^* = 0, \tag{29}$$

$$\nabla \cdot \mathbf{H} = 0, \tag{30}$$

$$\nabla \times \mathbf{H} = +\frac{1}{c}\frac{\partial \mathbf{E}^*}{\partial t}, \tag{31}$$

$$\nabla \times \mathbf{E}^* = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t}. \tag{32}$$

While the (free) field $\mathbf{E}^*$, undoubtedly, satisfies Maxwell equations, *it is not the consequence of Maxwell equations* (contrary to generally accepted point of view).[9]

To summarize this, we would like to recall a generally accepted point of view that due to the accelerated (or, in particular, oscillating) motion of the charges making up the radiating globally neutral source, the flux lines of the electric field *leave* the charges, *close* by themselves and form a *free*, progressively propagating (toward infinity), electromagnetic field.

Unfortunately, this reasoning is no more than words which are not supported by mathematical formulas. In this respect, it is well-known that in classical electrodynamics *there is no* any mathematical approach for describing a process of "*leaving*" and "*closing* (see, e.g., the expression 63.8 for the electric field obtained from Liénard-Wiechert potentials [2]). It is well-known fact that "Coulomb" part of the field as well as "accelerated" part cannot be described by the flux of lines that have no connection to the charge (in generally acepted interpretation these lines are closing near the surface of the charge and then leave the near region being already disconnected from the charge). In our work we show that the absence of this mechanism in the framework of conventional theory is not a mere coincidence or accident. As a matter of fact, according to the rigorous mathematical interpretation of Maxwell's equation (without any approximation) this mechanism cannot exist for the total electric field.

Thus, in the framework of Maxwell's theory, a free field can be understood only as a valid approximation for regions far from charges and current but not as an *adequate concept by itself.* Many physicist might not give importance to this fact (as the accustomed to work with approximation). Nevertheless, this subtle point is important to find out where this approximation (by the way, adopted also for quantum electrodynamics) is already not valid. Clarification of these points can give us additional information about limitations and hidden difficulties of classical electromagnetic theory (which, as

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[9] Any *nonelectric* zero-divergence field *also* satisfies Maxwell equations!

we know, recently is put very much in doubt[10]). These investigations will show us whether Maxwell theory may be improved *without* or *with* modification of its basic equations.

We turn now to some implications of our interpretation *without* postulating an existence of the free field $\mathbf{E}^*$.

3. IMPLICATIONS OF OUR INTERPRETATION

3.1 Isolated Charge-Free Region

Consider an isolated region $\mathcal{R}_0$ where no charges are present, i.e., $\mathcal{Q} = 0$, $\varrho = 0$ everywhere. Eq. (11) applies, so that $\mathbf{E} = 0$ everywhere in the whole space spanning $\mathcal{R}_0$. Assuming that Maxwell's equations are valid in $\mathcal{R}_0$ it follows that magnetic field $\mathbf{H}$ may still exist, because Maxwell's Eq. (6) is completely independent of ϱ. Indeed, in addition to the trivial solution $\mathbf{H} = 0$, many other solutions of $\nabla \cdot \mathbf{H} = 0$ are possible. For instance, $\mathbf{H} = H_x\mathbf{i} + H_y\mathbf{j} + H_z\mathbf{k}$ with $H_x = F_x(ct; y, z)$, $H_y = F_y(ct; x, z)$, $H_z = F_z(ct; x, y)$.

In a charge-free region Faraday's equation (8) reduces to $\partial\mathbf{H}/\partial t = 0$, hence $\mathbf{H}$ is time-independent. Our generic solution thus becomes $H_x = F(y, z)$, $H_y = F(x, z)$, and $H_z = F(x, y)$, where we have noted that in isotropic region there is no reason for the functional dependence to be different along arbitrary orientations.

Finally, Ampère's law (7) may lead (see Eqs. (19) and (27)) to

$$\nabla \times \mathbf{H} = \frac{4\pi}{c}\mathbf{j}_{\mathrm{mag}}, \tag{33}$$

where $\mathbf{j}_{\mathrm{mag}}$ may be some *magnetic* displacement current density. Eq. (33) does not impose further constraints onto $\mathbf{H}$, but rather defines the magnetic current $\mathbf{j}_{\mathrm{mag}}$. It may be immediately verified that the continuity condition $\nabla \cdot \mathbf{j}_{\mathrm{mag}} = 0$ is fulfilled by all $\mathbf{j}_{\mathrm{mag}}$ defined by Eq. (33). As an explicit example, let $H_x = F(y, z) = \sin[k(y + z)]$, et cyclicum. Then $j_x = (ck/4\pi)\{\cos[k(x + y) - \cos[k(x + z)]\}$, et cyclicum, where k is in inverse length units.

Summarizing, in a charge-free region described by ME no electric field is internally generated, but there *may* exist a time-independent magnetic background.

[10] See a brilliant review "Essay on Non-Maxwellian theories of electromagnetism," by V.V.Dvoeglazov [8].

3.2 Non-Isolated Charge-Free Region

Consider now a region $\mathcal{R}_0$ where no charges are present, $\mathcal{Q} = 0$, surrounded by a universe $\mathcal{U}$ where charges do exist. From the superposition principle, total electric field in the region is $\mathbf{E}(\mathcal{R}) = \mathbf{E}(\mathcal{R}_0) + \mathbf{E}(\mathcal{U}) = \mathbf{E}(\mathcal{U})$, where $\mathbf{E}(\mathcal{R}_0)$ denotes the field *internally* generated, and $\mathbf{E}(\mathcal{U})$ represents the field externally produced; from Eq. (11), $\mathbf{E}(\mathcal{R}_0) = 0$. Likewise, for the total magnetic field in the region, $\mathbf{H}(\mathcal{R}) = \mathbf{H}(\mathcal{R}_0) + \mathbf{H}(\mathcal{U})$, where $\mathbf{H}(\mathcal{R}_0)$ is time-independent (see the discussion in previous Sec. 3.1).

It is thus clear that the electric field $\mathbf{E}(\mathcal{R})$ existing inside a charge-free region *is not a free field*; rather, it is generated by charges outside the region. Of course, there is no contradiction with Gauss' law (10), which refers to $\mathbf{E}(\mathcal{U})$ entering and leaving the charge-free region.

3.3 The Simplest Charge-Neutral Universe

Consider a universe containing two equal charges of opposite sign. We can easily obtain from ME with $\varrho = 0$ different solutions $\{\mathbf{E}(\mathcal{U}), \mathbf{H}(\mathcal{U})\}$, depending upon the initial velocities and separation of the charges.

Consider now a phenomenon that was unknown to Maxwell: charge annihilation. What happens to the electric field $\mathbf{E}(\mathcal{U})$ if the charges meet to annihilate and form two photons? The obvious answer is nothing, the electromagnetic field $\{\mathbf{E}(\mathcal{U}), \mathbf{H}(\mathcal{U})\}$ continues its existence associated to the photons. None the less, there is a difficulty because we are now in situation of $\mathcal{Q} = 0$.[11]

So, in a universe populated by two photons there are several fundamental questions to answer. Firstly, do ME apply to them? Let us assume a positive answer. Then, secondly, are we in a charge-free or in a charge-neutral situation? Each possibility has different implications for the inner structure of photons. If photons do not contain charge at all, we are in a charge-free situation where the electric field has disappeared: $\mathbf{E}(photons) = 0$ (recall Eq. (11)). Hence, all information about the photons must be contained in the time-dependent magnetic field $\mathbf{H}(\mathcal{U})$. However, as discussed in Sec. 3.1 above, in a charge-free region $\mathbf{H}(\mathcal{R}_0)$ is time-independent, which means that the field $\mathbf{H}(\mathcal{U})$ is frozen in time at the moment of annihilation.

Alternatively, if we are in a charge-neutral situation, then the electromagnetic field $\{\mathbf{E}(\mathcal{U}), \mathbf{H}(\mathcal{U})\}$ may continue to exist associated now to the two photons. But then, it means that inside each charge-neutral photon there *must exist* at least a hidden dipole! This interpretation nicely blends with the current view from field theory

[11] See point (b) in Sec. 2

that attaches electric dipole fields to photons.

4. CONCLUDING REMARKS

In this paper we argued that a rigorous application of Gauss' law to the solution of Maxwell's equations leads to the identification of two situations: *charge-free* and *charge-neutral*. This immediately implies that electric *free field* is not *a consequence* of Maxwell's equations: one only may postulate it.

In an isolated charge-free vacuum, electric field does not exist, but there *may* exist a time-independent background magnetic field. A consideration of the simplest charge-neutral universe leads to some interesting conjectures regarding the inner structure of photons.

Concerning the Sec. 3.3: There we intended to underline that if the free electric field cannot be considered as direct consequence of Maxwell equation (without generally accepted approximation), then this can lead to subtle but drastic reconsideration of approximations adopted in *quantum* electrodynamics. This can give rise to the opinion that classical as well as quantum electrodynamics might become compatible with the photon finite mass conception which, as we know, independently have arisen in different modern approaches on quantum theory of light (de Broglie, etc).

In concluding this letter, we would like to emphasize that in this work we considered only a classical theory and every comment on quantum electrodynamics had a purely suggestive character.

ACKNOWLEDGMENTS

The authors are indebted to Profs. V. Dvoeglazov [one of us (H.M.) thanks him for his kind invitation to visit the School of Physics of the Zacatecas University in the context of the CONACyT project No. 0270P-E], S. Vlaev, G. Kalberman, O. Guzman, and L.-M. Gaggero for helpful discussion and critical comments. We are especially indebted to the Editor for his choice of the referees: Their valuable remarks helped us improve our reasoning considerably.

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