# The Triple Higgs Boson Self-Coupling at Future Linear $e^+e^-$ Colliders Energies: ILC and CLIC

Alejandro GUTIÉRREZ-RODRÍGUEZ<sup>\*</sup>, Angeles HERNÁNDEZ-RUÍZ<sup>1</sup>, Oscar SAMPAYO<sup>2</sup>, Andrei CHUBYKALO, and Augusto ESPINOZA-GARRIDO

Facultad de Física, Universidad Autónoma de Zacatecas, Apartado Postal C-580, 98060 Zacatecas, Zacatecas, México <sup>1</sup>Instituto de Física, Universidad Autónoma de San Luis Potosí, 78000 San Luis Potosí, SLP, México

<sup>2</sup>Departamento de Física, Universidad Nacional del Mar del Plata, Funes 3350, (7600) Mar del Plata, Argentina

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We analyzed the triple Higgs boson self-coupling at future  $e^+e^-$  colliders energies, with the reactions  $e^+e^- \rightarrow b\bar{b}HH$ ,  $t\bar{t}HH$ . We evaluate the total cross-sections for both  $b\bar{b}HH$  and  $t\bar{t}HH$ , and calculate the total number of events considering the complete set of Feynman diagrams at tree-level. We vary the triple coupling  $\kappa \lambda_{3H}$  within the range  $\kappa = -1$  and +2. The numerical computation is done for the energies expected to be available at a possible Future Linear  $e^+e^-$  Collider with a center-of-mass energy 800, 1000, 1500 GeV and a luminosity  $1000 \text{ fb}^{-1}$ . Our analysis is also extended to a center-of-mass energy 3 TeV and luminosities of  $1000 \text{ and } 5000 \text{ fb}^{-1}$ . We found that for the process  $e^+e^- \rightarrow b\bar{b}HH$ , the complete calculation differs only by 3% from the approximate calculation  $e^+e^- \rightarrow ZHH(Z \rightarrow b\bar{b})$ , while for the process  $e^+e^- \rightarrow t\bar{t}HH$ , the expected number of events, considering the decay products of both t and H, is not enough to obtain an accurate determination of the triple Higgs boson self-coupling.

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#### 1. Introduction

The Higgs  $boson^{1-3}$  plays an important role in the Standard Model  $(SM)^{4-6}$  because it is responsible for generating the masses of all elementary particles (leptons, quarks, and gauge bosons). However, the Higgs-boson sector is the least tested in the SM, in particular the Higgs boson self-interaction.

The search for Higgs bosons is one of the principal missions of present and future high-energy colliders. The observation of this particle is of major importance for the present understanding of fundamental particle interactions. Indeed, in order to accommodate the well established electromagnetic and weak interaction phenomena, the existence of at least one isodoublet scalar field to generate fermion and weak gauge bosons masses is required. Despite previous success in explaining the present data, the SM cannot be completely tested before this particle has been experimentally observed and its fundamental properties studied.

The triple and quartic Higgs boson couplings<sup>7-14)</sup>  $\lambda_{3H}$  and  $\lambda_{4H}$  are defined through the potential:

$$V(H) = \frac{M_{\rm H}^2}{2} H^2 + \frac{M_{\rm H}^2}{2v} H^3 + \frac{M_{\rm H}^2}{8v^2} H^4, \qquad (1)$$

where the triple and quartic couplings of the Higgs field H are given by

$$\lambda_{\rm 3H} = \frac{3M_{\rm H}^2}{M_Z^2} \lambda_0,\tag{2}$$

$$\lambda_{4\rm H} = \frac{3M_{\rm H}^2}{M_2^4} \lambda_0^2.$$
 (3)

To obtain these expressions we assumed the normalization employed in refs. 9–14, where  $\lambda_0 = M_Z^2/v$ .

In the SM, we obtain  $M_{\rm H} = \sqrt{2\lambda}v$  as the simple relationship between the Higgs boson mass  $M_{\rm H}$  and the selfcoupling  $\lambda$ , where v = 246 GeV is the vacuum expectation value of the Higgs boson. The triple vertex of the Higgs field H is given by eq. (2) and a measurement of  $\lambda_{3\rm H}$  in the SM can determine  $M_{\rm H}$ . An accurate test of this relationship may reveal the extended nature of the Higgs sector. The measurement of the triple Higgs boson coupling is one of the most important goals of Higgs physics in a future  $e^+e^$ linear collider experiment. This would provide the first direct information on the Higgs potential responsible for electroweak symmetry breaking.

The triple Higgs boson self-coupling can be measured directly in pair-production of Higgs particles at hadron and high-energy  $e^+e^-$  linear colliders. Several mechanisms that are sensitive to  $\lambda_{3H}$  can be exploited for this task. Higgs pairs can be produced through double Higgs-strahlung of W or Z bosons,<sup>9–21)</sup> WW or ZZ fusion;<sup>8,22–25)</sup> moreover, through gluon–gluon fusion in pp collisions<sup>26–29)</sup> and high-energy  $\gamma\gamma$  fusion<sup>8,30)</sup> at photon colliders. The two main processes at  $e^+e^-$  colliders are double Higgs-strahlung and WW fusion:

double Higgs-strahlung: 
$$e^+e^- \rightarrow ZHH$$
,  
WW double-Higgs fusion:  $e^+e^- \rightarrow \bar{\nu}_e \nu_e HH$ . (4)

The ZZ fusion process of Higgs boson pairs is suppressed by an order of magnitude because the electron-Z coupling is small. The more suitable reaction in  $e^+e^-$  colliders to measure the triple couplings in the range of the theoretically preferred Higgs mass O (100 GeV) is the double Higgsstrahlung process  $e^+e^- \rightarrow ZHH$ . Operating at center-ofmass energy  $\sqrt{s}$  from 500 GeV up to about 1 TeV, the International Linear Collider (ILC)<sup>31,32)</sup> can measure the *HHZ* production cross-section (about 0.1-0.2 fb) if the Higgs boson mass is  $M_{\rm H} = 120 \,{\rm GeV}$ .<sup>33)</sup> When the center-ofmass energy  $\sqrt{s}$  exceeds 1 TeV, the  $e^+e^- \rightarrow \bar{\nu}_e v_e HH$  mode becomes sizeable and it is possible to measure the triple

<sup>\*</sup>E-mail: alexgu@planck.reduaz.mx



Fig. 1. Feynman diagrams at tree-level for  $e^+e^- \rightarrow b\bar{b}HH$ .

Higgs self-coupling  $\lambda_{3H}$  by using this process. Therefore, in the first stage of the ILC ( $\sqrt{s} < 1 \text{ TeV}$ ),  $e^+e^- \rightarrow ZHH$ is the most promising channel to measure the triple Higgs self-coupling  $\lambda_{3H}$ . In this process, the final state of two Higgs may be generated by the interchange of a virtual Higgs in such a way that this process is sensitive to the triple coupling *HHH* in the Higgs potential. It is necessary to include four-body processes with heavy fermions f,  $e^+e^- \rightarrow f\bar{f}HH$ , in which the SM Higgs boson is radiated by a  $b(\bar{b})$  quark at future  $e^+e^-$  colliders<sup>34-41</sup> with a c.m. energy in the range of 800 to 1500 GeV, as in the case of the ILC<sup>31,32</sup> and Compact Linear Collider (CLIC)<sup>42</sup> machines, in order to know its impact on the three-body channel and also to search for new relations that may have a clear signature of the Higgs boson production.

The Higgs coupling with top quarks, the largest coupling in the SM, is directly accessible in the process where

the Higgs boson is radiated off top quarks,  $e^+e^- \rightarrow t\bar{t}HH$ . This process depends on the Higgs boson triple selfcoupling, which could lead us to obtain the first nontrivial information on the Higgs potential. We are interested in finding regions that could allow the observation of the bbHH and  $t\bar{t}HH$  processes at future linear  $e^+e^$ colliders energies: ILC and CLIC. In the process  $e^+e^- \rightarrow$  $b\bar{b}HH$ , the set of figures shown for the  $b\bar{b}HH$  final state includes the ZHH process with  $Z \rightarrow b\bar{b}$ . We found that the results for the complete calculation  $e^+e^- \rightarrow b\bar{b}HH$ and for the approximate  $e^+e^- \rightarrow ZHH$  with an on-shell Z decay to  $b\bar{b}$ , differ only at the 3% level in the examined kinematic range. We consider the complete set of Feynman diagrams at tree-level (Figs. 1 and 2) and use the CALCHEP<sup>43)</sup> packages to evaluate the amplitudes and cross-section of the processes  $e^+e^- \rightarrow b\bar{b}HH$  and  $e^+e^- \rightarrow$ tīHH.



Fig. 2. Feynman diagrams at tree-level for  $e^+e^- \rightarrow t\bar{t}HH$ .

This paper is organized as follows: In §2, we study the triple Higgs boson self-coupling through the processes  $e^+e^- \rightarrow b\bar{b}HH$  and  $e^+e^- \rightarrow t\bar{t}HH$  at future linear  $e^+e^-$  colliders energies and, finally, we summarize our results in §3.

# 2. Cross-Section of the Higgs Boson Pairs Production with Triple Self-Coupling

In this section we present numerical results for  $e^+e^- \rightarrow b\bar{b}HH$  and  $e^+e^- \rightarrow t\bar{t}HH$  with double Higgs boson production. We carry out the calculations using the Standard Model framework at future linear  $e^+e^-$  colliders energies. We use the CALCHEP<sup>43</sup> packages for calculations of the matrix elements and cross-sections. These packages provide automatic computation of the cross-sections and distributions in the SM as well as their extensions at tree-level. We consider the high energy stage of a possible future linear  $e^+e^-$  collider with  $\sqrt{s} = 800$ , 1000, 1500 GeV and the designed luminosity 1000 fb<sup>-1</sup>.

#### 2.1 Triple Higgs boson self-coupling via $e^+e^- \rightarrow b\bar{b}HH$ , $t\bar{t}HH$

In order to illustrate our results for the sensitivity to the HHH triple Higgs boson self-coupling, we show the  $\kappa$ dependence of the total cross-section for  $e^+e^- \rightarrow b\bar{b}HH$ in Fig. 3 and for  $e^+e^- \rightarrow t\bar{t}HH$  in Fig. 4. We consider one representative value of the Higgs boson mass,  $M_{\rm H} = 130$ GeV, with a center-of-mass energy of  $\sqrt{s} = 800$ , 1000, 1500 GeV, varying the triple coupling  $\kappa \lambda_{3H}$  within the range  $\kappa = -1$  and +2. In both cases, the cross-section is sensitive. to the value of the triple coupling. The sensitivity to  $\lambda_{3H}$ increases with the collider energy, reaching a maximum at  $\sqrt{s} \sim 600 \,\text{GeV}$  for the *bbHH* channel and at  $\sqrt{s} \sim$ 1200 GeV for the ttHH channel (Figs. 5 and 6). As an indicator of the order of magnitude, in Tables I-III we present the number of events of Higgs bosons expected for several Higgs boson masses, center-of-mass energy and  $\kappa$ values and for  $1000 \, \text{fb}^{-1}$  luminosity (of course, we have



Fig. 3. Variation of the cross-section  $\sigma(b\bar{b}HH)$  with the modified triple coupling  $\kappa \lambda_{3\rm H}$  at a collider energy of  $\sqrt{s} = 800$ , 1000, 1500 GeV and  $M_{\rm H} = 130$ .



Fig. 4. The same as in Fig. 3, but for the process  $e^+e^- \rightarrow t\bar{t}HH$ .



Fig. 5. The dependence of the cross-section on center-of-mass energy  $\sqrt{s}$  for two fixed Higgs masses  $M_{\rm H} = 110$ , 130 GeV. The variation of the cross-section for modified triple couplings  $\kappa \lambda_{3\rm H}$  is indicated by the solid and dot-dashed lines.



Fig. 6. The same as in Fig. 5, but for the process  $e^+e^- \rightarrow t\bar{t}HH$ .

Table 1. Total production of Higgs boson pairs in the SM for  $\mathcal{L} = 1000 \, \mathrm{fb^{-1}}$  and  $\kappa = 0.5$ .

Total production of Higgs boson pairs <i>M</i> <sub>H</sub> (GeV)	$e^+e^- \rightarrow b\bar{b}HH(t\bar{t}HH)$ $\kappa = 0.5$		
	$\sqrt{s} = 800 \mathrm{GeV}$	$\sqrt{s} =$ 1000 GeV	$\sqrt{s} =$ 1500 GeV
110	20 (11)	16 (18)	10 (17)
130	17 (5)	14 (11)	9 (13)
150	14 (2)	12 (6)	9 (9)
170	11 ()	11 (4)	8 (7)
190	9 ()	10 (2)	8 (5)

Table II. Total production of Higgs boson pairs in the SM for  $\mathcal{L} = 1000 \, \text{fb}^{-1}$  and  $\kappa = 1$  (SM).

Total production of Higgs boson pairs	$e^+e^- \rightarrow b\bar{b}HH(t\bar{t}HH)$ $\kappa = 1$ (SM)		
M <sub>H</sub> (GeV)	$\sqrt{s} =$ 800 GeV	$\sqrt{s} =$ 1000 GeV	$\sqrt{s} =$ 1500 GeV
110	23 (13)	18 (21)	12 (19)
130	21 (5)	17 (13)	11 (14)
150	18 (2)	16 (8)	10 (11)
170	15 ()	14 ()	10 (8)
190	13 ()	13 ()	10 (6)

Table III. Total production of Higgs boson pairs in the SM for  $\mathcal{L} = 1000 \, \text{fb}^{-1}$  and  $\kappa = 1.5$ .

Total production of Higgs boson pairs	$e^+e^- \rightarrow b\bar{b}HH(t\bar{t}HH)$ , $\kappa = 1.5$		
M <sub>H</sub> (GeV)	$\sqrt{s} =$ 800 GeV	$\sqrt{s} =$ 1000 GeV	$\sqrt{s} =$ 1500 GeV
110	28 (15)	21 (24).	13 (20)
130	26 (6)	21 (15)	13 (16)
150	23 (3)	20 (9)	13 (13)
170	20 ()	18 (5)	13 (10)
190	17 ()	17 (3)	13 (8)

multiplied by the corresponding Branching Ratios to obtain the observable number of events). In particular, if we consider the  $H \rightarrow b\bar{b}$  decay for  $M_{\rm H} < 130$  GeV, there is some possibility to detect the  $e^+e^- \rightarrow b\bar{b}HH$  process. In this region, the number of events is small but sufficient to detect  $e^+e^- \rightarrow b\bar{b}HH \rightarrow b\bar{b}b\bar{b}b\bar{b}$ , in which the  $BR(H \rightarrow b\bar{b}) \sim 0.6$  and the background for 6b-jet are small.

For the  $e^+e^- \rightarrow t\bar{t}HH$  process, the center-of-mass energy 1000 GeV and  $M_{\rm H} < 130$  GeV is the most favorable, but the Branching Ratios for the four decay modes make this process very small.

For the center-of-mass energy  $\sqrt{s}$  from 800 GeV up to about 1 TeV, the production of  $b\bar{b}HH$  and  $t\bar{t}HH$  in the intermediate mass range of the *H* mass is significant and all the final states can be identified without large momentum loss. When the c.m. energy  $\sqrt{s}$  exceeds 1 TeV, the crosssection decreases and therefore in the first stage of a future ILC ( $\sqrt{s} \le 1 \text{ TeV}$ ),  $e^+e^- \rightarrow b\bar{b}HH$  and  $e^+e^- \rightarrow t\bar{t}HH$  are important channels to measure the triple Higgs boson selfcoupling.



Fig. 7. Contour plot for the number of events of the process  $e^+e^- \rightarrow b\bar{b}HH$  as a function of  $M_{\rm H}$  and  $\sqrt{s}$ . The variation of the number of events for modified triple couplings  $\kappa \lambda_{3\rm H}$  is indicated for  $\kappa = 0.5$ , 1 (SM), 1.5.

Finally, we include a contour plot for the number of events of the studied processes as a function of  $M_{\rm H}$  and  $\sqrt{s}$  with  $\kappa = 0.5$ , 1 (SM), 1.5 in Figs. 7 and 8. These contours are obtained from Tables I–III.

## 2.2 Triple Higgs boson self-coupling via $e^+e^- \rightarrow b\bar{b}HH$ , tīHH at CLIC energies

In this subsection we analyze the triple Higgs selfcoupling  $\lambda_{3H}$  via the processes  $e^+e^- \rightarrow b\bar{b}HH$ ,  $t\bar{t}HH$  for energies expected at the CLIC.<sup>42)</sup> Figures 9 and 10 show the total cross-section for the double Higgs-strahlung in  $e^+e^$ collisions,  $e^+e^- \rightarrow bbHH$ , ttHH as a function of  $M_{\rm H}$  for the c.m. energy of  $\sqrt{s} = 3 \text{ TeV}$  and  $\kappa = 0.5, 1$  (SM), 1.5. The effects of a variation of the triple coupling by 50% from its SM value are shown in these figures. The production cross-section is of the order of a fraction of a femtobarn  $(0.005 \text{ fb for } b\bar{b}HH \text{ and } 0.008 \text{ fb for } t\bar{t}HH)$  when it is not overly suppressed by phase-space and it is mediated by s channel gauge boson exchange. From these figures, we observe that the total cross-sections of both processes are too small because their order of magnitude is smaller than that for the case of  $\sqrt{s} = 800$ , 1600 GeV, as indicated in refs. 37-41.

As in §2.1, we show the  $\kappa$  dependence of the total crosssection for  $e^+e^- \rightarrow b\bar{b}HH$ ,  $t\bar{t}HH$  in Figs. 11 and 12. We consider one representative value of the Higgs boson mass,



Fig. 8. The same as in Fig. 7, but for the process  $e^+e^- \rightarrow t\bar{t}HH$ .



Fig. 9. The cross-section for the double Higgs-strahlung via  $e^+e^- \rightarrow b\bar{b}HH$ , at a c.m. energy of  $\sqrt{s} = 3 \text{ TeV}$  as a function of  $M_{\rm H}$  with  $\kappa = 0.5$ , 1 (SM), 1.5. The effects of a variation of the triple coupling by 50% from its SM value are shown.

 $M_{\rm H} = 130 \,{\rm GeV}$ , and center-of-mass energy  $\sqrt{s} = 3 \,{\rm TeV}$ , varying the triple coupling  $\kappa \lambda_{3\rm H}$  within the range  $\kappa = -1$ and +2. In both cases, the production cross-sections are also too small because their order of magnitude is smaller than that for the case of  $\sqrt{s} = 800$ , 1500 GeV and  $M_{\rm H} = 130$ GeV, as is illustrated in Figs. 3 and 4 of §2.1.



Fig. 10. The same as in Fig. 9, but for the process  $e^+e^- \rightarrow t\bar{t}HH$ .



Fig. 11. Variation of the cross-section  $\sigma(b\bar{b}HH)$  with the modified triple coupling  $\kappa \lambda_{3H}$  at a collider energy of  $\sqrt{s} = 3 \text{ TeV}$  and  $M_{\rm H} = 130$ .





Table IV. Total production of Higgs boson pairs in the SM for  $\sqrt{s} = 3$  TeV and  $\mathcal{L} = 1000$  fb<sup>-1</sup>.

Total production of Higgs boson pairs M <sub>H</sub> (GeV)	$e^+e^- \rightarrow b\bar{b}HH(t\bar{t}HH),  \sqrt{s} = 3 \text{ TeV}$		
	$\kappa = 0.5$	$\kappa = 1$ (SM)	$\kappa = 1.5$
110	5 (7)	5 (8)	5 (8)
130	5 (6)	5 (7)	5 (7)
150	4 (5)	5 (6)	5 (7)
170	4 (4)	5 (5)	5 (6)
190	4 (4)	5 (4)	6 (6)

Table V. Total production of Higgs boson pairs in the SM for  $\sqrt{s} = 3$  TeV and  $\mathcal{L} = 5000 \, \text{fb}^{-1}$ .

Total production of Higgs boson pairs $M_{\rm H}$ (GeV)	$e^+e^- \rightarrow b\bar{b}HH(t\bar{t}HH),  \sqrt{s} = 3 \mathrm{TeV}$		
	$\kappa = 0.5$	$\kappa \simeq 1 \text{ (SM)}$	$\kappa = 1.5$
110	24 (35)	25 (39)	27 (42)
130	23 (29)	24 (33)	26 (37)
150	22 (26)	24 (28)	27 (33)
170	21 (21)	24 (25)	27 (30)
190	21 (18)	24 (22)	28 (28)

Finally, in Tables IV and V we present the Higgs boson number of events for several Higgs boson masses,  $\kappa$  values, luminosities of 1000 and  $5000 \, \text{fb}^{-1}$  and center-of-mass energy  $\sqrt{s} = 3 \text{ TeV}$  (of course, we have multiplied by the corresponding Branching Ratios to obtain the observable number of events). It is clear from Figs. 9-12 and Table IV that it would be difficult to obtain a clear signal of the processes  $e^+e^- \rightarrow b\bar{b}HH$ ,  $t\bar{t}HH$  at energies of a future linear collider such as CLIC, after having considered the background, except for  $\sqrt{s} = 3 \text{ TeV}$  and very high luminosity  $(\mathcal{L} = 5000 \, \text{fb}^{-1})$  as is shown in Table V. However, for the center-of-mass energy of CLIC, the WW double Higgs fusion process, 7,8,33 which increases with rising  $\sqrt{s}$ , can be exploited by larger energies and luminosities, and would be the preferred mechanism to measure the triple Higgs selfcoupling  $\lambda_{3H}$ .

#### 3. Conclusions

 $e^+e^-$  linear colliders represent a possible opportunity for triple Higgs boson self-coupling analysis. Therefore, we have analyzed the triple Higgs boson self-coupling at future  $e^+e^-$  collider energies with the reactions  $e^+e^- \rightarrow b\bar{b}HH$ and  $e^+e^- \rightarrow t\bar{t}HH$ . The ILC has access to the triple Higgs boson self-coupling through the double Higgs production processes  $e^+e^- \rightarrow ZHH$  and  $e^+e^- \rightarrow \nu\bar{\nu}HH$ .<sup>7-14,17-21,33)</sup> Although the cross section for  $e^+e^- \rightarrow ZHH$  with intermediate Higgs boson mass is only about 0.1-0.2 fb for  $\sqrt{s}$  < 1 TeV, the measurement of the Higgs self-coupling  $\lambda_{3H}$  at  $e^+e^-$  colliders can be significantly improved. For example, in ref. 33, Castanier et al. concluded that a precision of about 10% on the total cross-section for  $e^+e^- \rightarrow ZHH$  can be achieved, leading to a relative error on  $\lambda_{3H}$  of 18% with the help of high integrate luminosity  $\mathcal{L} = 2ab^{-1}$  after performing the detailed simulations of signal and background process at the TESLA.44) Other simulations<sup>17-20)</sup> demonstrate that the Higgs self-coupling

 $\lambda_{3H}$  can be extracted more accurately by using some discriminating kinematic variables, namely the invariant mass of the HH system and the extraction of the Higgs selfcoupling  $\lambda_{3H}$  can be further improved to an accuracy of 8% and even better in multi-TeV  $e^+e^-$  collisions.<sup>21)</sup> Therefore, to determine the triple Higgs boson self-coupling  $\lambda_{3H}$  via the process  $e^+e^- \rightarrow b\bar{b}HH$  and due to the cross-section difference of 3% between  $ZHH(Z \rightarrow bb)$  and bbHH, the conclusions for the precision in the determination of  $\lambda_{3H}$  are not significantly modified. That is to say, we expect that the results for the complete computation of the process  $e^+e^- \rightarrow$ bbHH should not alter the conclusions of the previous computation<sup>17-20,33)</sup> and thus the same background analysis for  $e^+e^- \rightarrow ZHH(Z \rightarrow b\bar{b})$  remains valid for  $e^+e^- \rightarrow$ bbHH. Examination of variables sensitive to the triple Higgs boson vertex and the availability of high luminosity will allow testing of the Higgs potential structure at future linear  $e^+e^-$  colliders (in the case of the Minimal Supersymmetric extension of the Standard Model (MSSM) with large  $\tan \beta$ , the *bbHH* channel may be significantly enhanced). On the other hand, for the  $t\bar{t}HH$  final state, we found a major number of events (Table V) to difference of the bbHH channel, but after considering the decay products of both the Higgs boson (H) and the top quark (t) to b quarks, the expected final number of events will be very small. Finally, the study of these processes is important and could be useful to probe the triple Higgs boson selfcouplings  $\lambda_{3H}$  given the following conditions: very high luminosity, excellent b tagging performance, and center-ofmass energy in the range  $\sqrt{s} = 800 - 1000 \,\text{GeV}$ , which is the most favorable colliding energy for bbHH and  $t\bar{t}HH$ production and for the lightest Higgs boson mass in the range  $M_{\rm H} = 110-130 \,{\rm GeV}$ . In addition, these results have never been reported in the literature before and could be of relevance for the scientific community.

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