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CONTRIBUTIONS / CHAPTERS

Fast flame temperature estimation using a point diffraction interferometer and non-negative least square method

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Abstract

Some of the interferometry methods proposed for flame temperature measurements from its projection could be complex and demand so much computing time. Assuming a circular symmetric and smooth flame temperature distribution, it is possible to use a linear combination of Gaussian functions with weights constrained to non-negative values.

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Introduction

The temperature of an object is a measure of the thermal energy. It represents the total internal energy of the object. In general, temperature is determined by measuring an optical, mechanical or electrical property of a material that varies with temperature. Temperature measurements using mechanical and electrical techniques are effective when the object is solid with homogeneous temperature and the sensor is in thermal equilibrium. For non-solid objects like flames, these techniques present several problems, Yilmaz et al [1], e.g., a great number of measurements are required to obtain the total volumetric temperature of the flame and the its temperature distribution is modified by the sensors. On the other hands , optical methods do not perturb the flame temperature and also they allow to have a bigger set of measurements.
Several optical methods are based on color [2, 3], infrared [4], and interferometric [2, 5, 6] techniques. Interferometric methods are widely used to measure deformation, tension, temperature, etc [7, 8] in a non-invasive and non-destructive way. Such magnitudes produce a frequency modulated fringe pattern called interferogram. Demodulation and phase inversion processes are needed to estimate the desired physical magnitude. Most of the phase recovering methods are based on the Fourier Transform [9], phase shifting [10] or regularization [11-13] techniques. Techniques for phase recovery such as Fourier based produces a wrapped phase in the interval $(-\pi, \pi]$. To unwrap this phase, path dependent algorithms [14] can be applied. Ghiglia et. al. shows a simple test for path dependence [15]. A robust alternative for many cases is the least-squares solution, which is described in matrix form by Hunt [16]. Another robust algorithm to find a solution in the presence of path-integral phase inconsistencies using the cosine transform is that proposed by Ghiglia and Romero [17]. Optical tomography is a method used to obtain the spatial distribution of the refraction index of a phase object (PO) on its non refractive index (refractionless limit) from one or more projections. For the case of radially symmetrical phase objects only one projection is necessary to reconstruct the refractive index distribution. This projection is formed by a set of summa rays (Figure 1). Tomographic reconstruction methods can be grouped into two categories: back projection methods and algebraic methods (ART) [18, 19, 20, 21]. In the algebraic method, the projections are a linear transformation of the cross sections of the object, i.e., a linear system given by the vector solutions (projection of each section), a transformation matrix and a vector of unknowns (cross section of the linear system phase object) [7, 18, 19, 20, 21]. The number of unknowns can be reduced assuming the temperature distribution in each section can be estimated by a linear combination of basis functions. In this paper, we present a simple and rapid method for measuring the temperature of a flame, using a point diffraction interferometer and a set of basis functions.

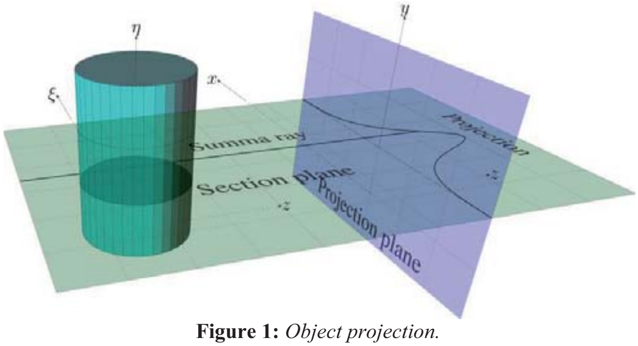


Figure 1: *Object projection.*

Base theory

Interferometry

Interferometry techniques are used to measure physical quantities [1, 7] such as temperature, pressure, tension or deformation associated to the refraction index of a object. A common objective of these methods is to produce a fringe pattern modulated by variations of these magnitudes. An interferogram is described mathematically by

$$I(x,y)=a(x,y)+b(x,y)\cos[2\ \ f_0x+\ \ (x,y)]\ \ _m(x,y)+\ \ _a(x,y), \tag{1.1}$$

where (x, y) are spatial coordinates, a(x, y) is the background illumination, b(x, y) is the amplitude modulation, $\phi(x, y)$ is the phase associated to the refraction index, f_0 is the carrier frequency [9], and $\eta_m(x, y)$ and $\eta_a(x, y)$ are the multiplicative and additive noises, respectively; in the case of Speckle Pattern Interferometry (SPI) [21] or single path interferometry [7], the noise is multiplicative. For higher levels of noise it is necessary the use of a filter to preserve fringes. In many cases $a(\cdot)$ and $b(\cdot)$ are considered as constants when they vary slowly. For interferogram with no carrier $(f_0 = 0)$ the interferogram can be rewritten as

$$I(x,y)=\cos[\ \ (x,y)]\ \ _m(x,y)+\ \ _d(x,y), \tag{1.2}$$

The optical path length (OPL) of one ray through a transparent medium is described by

$$=\ \ _C nds \tag{1.3}$$

OPL is along path C. When refraction is not significant, the path can be approximated by a straight line. If the beam is propagated along the *z* axis, as it is shown in Figure 2, the OPL can be expressed as:

$$(\ \ )=\ \ _C n(\ \ )dz \tag{1.4}$$

The optical path difference (OPD) $\Delta(\cdot)$ is given by

$$(\ \ )=\int_C[n(\ \ )\ \ n_0]\,dz, \tag{1.5}$$

where η_0 is the refraction index of the medium. The OPD is related to the phase ϕ in Equation 1.2 by:

$$(x)=\frac{2}{\ \ }(\ \ ) \tag{1.6}$$

For circularly symmetric optical paths (OP), Equation 1.6 can be expressed in terms of Abel Transform [7], *A{·} by*

$$(\ ) = A\left\{n(r)\quad n_0\right\} = A\left\{n_d(r)\right\} = 2\int^{+\infty} \frac{n_d(r)}{\sqrt{r^2 \quad ^2}}dr \tag{1.7}$$

where r is given by $\sqrt{\ ^2 + z^2}$.
One of the simplest approaches to find Abel transform of a circularly symmetric discrete function ($n_k = n(k)$, $k \in$), is by a linear combination of rings of width Δr and height f_k. The discrete Abel Transform, $A\{\cdot\}$, can be obtained by

$$N_k = A_d\{n_k\} = 2 \sum_{k=1} n_k \int_{r_k}^{r_k+1} \frac{r}{\left(r^2 \quad k^2\right)^{\frac{1}{2}}}dr \tag{1.8}$$

Solving the integral we obtain:

$$\frac{N_k}{2\ \ r} = \sum_{k=i} A_{i,k} n_k \tag{1.9}$$

where $A_{i,k} = \left[(k+1)^2 \quad i^2\right]^{\frac{1}{2}} \quad (k^2 + i^2)^{\frac{1}{2}}$.

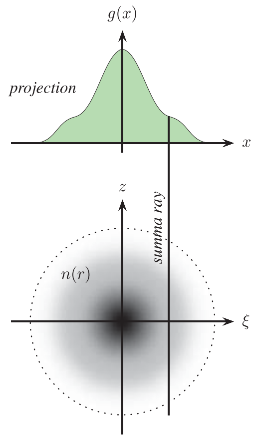


Figure 2: *Cross section object projections. OPL is calculated along the straight line.*

Function approximation using base functions

The purpose of interpolation is to obtain a function which best fits a set of points using a predefined cost function. The approximation process can be set as follows: Given a set of points $\left\{(x_i, y_i) \middle| x_i, y_i \quad , i \quad ^+\right\}$ find a function, f(x) such that

$$\min \left\| y_i \quad f(x_i) \right\|^2 \tag{1.10}$$

which f(x) could be a linear combination of basis functions

$$f(x) \quad \sum_j w_i g_j(x_i), j \quad ^+ \tag{1.11}$$

where the set of weights $\{w_j | w_j \ \}$ are those which optimize Equation 1.12.

$$\min_{w_i} \ \| y_i \quad w_j g_j(x_i) \|^2 \ , j \ ^{+} \qquad (1.12)$$

Point Diffraction Interferometer

In Figure 3, the interferometer of common path uses a diffractor element to measure the wavefront [22]. The Point diffraction interferometer (PDI) is located at the focus L_2. The PDI is a thin optical disc half the diameter of an Airy Disk, equal to 1.22 λ *f#*, where λ is wavelength, *f#* is the incident wavefront numeric aperture. The disk modulates the transmitted beam amplitude and phase. The PDI generates a synthetic wavefront superimposed to the original wavefront [5, 22, 23]. The basic idea of the PDI is shown in Figure 4, where the reference wavefront and the object produce an interferogram.

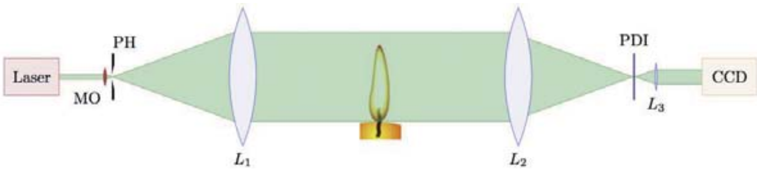


Figure 3: *Interferometer setup.*

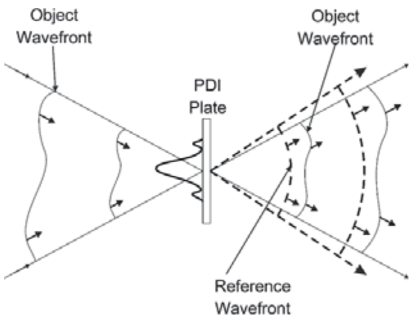


Figure 4: *PDI Incident (object) and transmitted wavefront (Reference and Object).*

Proposed reconstruction method

If PO is a smooth phase object, then, its refraction index (n(r)) can be approximated by a linear combination of k basis functions defined by Equation 1.11 and Equation 1.7 can be rewritten as

$$(\ ) = A\left\{\sum_k w_k f_k(r)\right\} = 2\int^{\infty} \frac{\sum_k w_k f_k(r)}{\sqrt{r^2 \quad ^2}}$$

$$= \sum_k w_k A\{f_k(r)\} = \sum_k w_k F_k(x) \qquad (1.13)$$

or in matrix form:

$$? = \mathbf{Fw} \tag{1.14}$$

One choice for the set of basis functions is a set of Gaussians. The positions of the gaussians can be evenly distributed on an interval L. The width σ of the Gaussian functions *f* may be determined by the following relationship

$$? = \frac{L}{2d+(n_g \quad 1)s} \tag{1.15}$$

where *d* is the distance of the lower limit to the center of the first Gaussian, *s* is the separation between gaussians (depending on σ) and n_g is the number of Gaussians (see Figure 5).

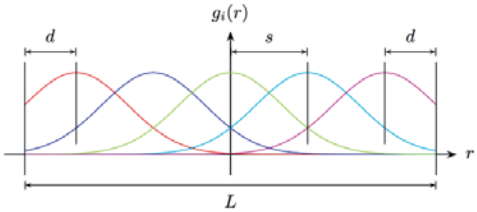


Figure 5: *Uniformly distributed (over an interval L) basis functions*

The optimal weights w* that fulfill Equation 1.14, produce oscillations that make $n_d(r) < 0$. Therefore, it is necessary to limit the solution to weights, w_k, equal or greater than zero, i.e.

$$\min_{?w} \left\| \quad \mathbf{Fw} \right\|^2, ??s.?t. w_k \geq 0 \tag{1.16}$$

To find the solution of Equation 1.16 the non-negative least square method can be used.

Results

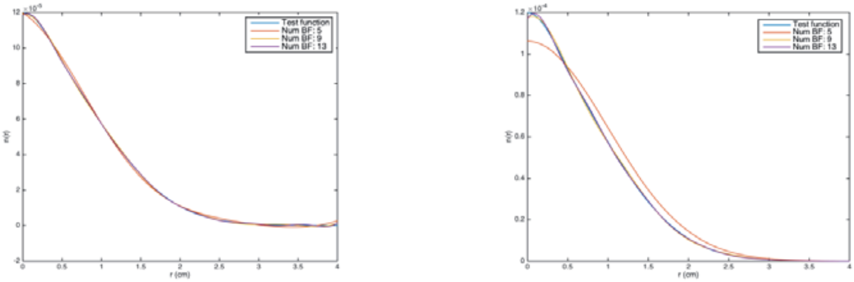
To show the reconstruction quality, a test function is used. This function is expressed as

$$n_t(r) = \quad 10^{\quad 4}\left[\exp\left(\quad 5r^2\right) + \exp\left(\quad \frac{5}{9}r^2\right)\right] \tag{1.17}$$

Figure 6 shows some constrained and non-constrained approximations. To compare results, in both approximations we use different number of basis functions (BF). We found that the constrained approximation has a very good fitting to the test function when it uses a linear combination of 5 basis functions; with a higher number of BF, error change is not significant. Also we can see that the non-constrained approximations goes below the value of "0", such thing does not happen in the case of the constrained approximation.

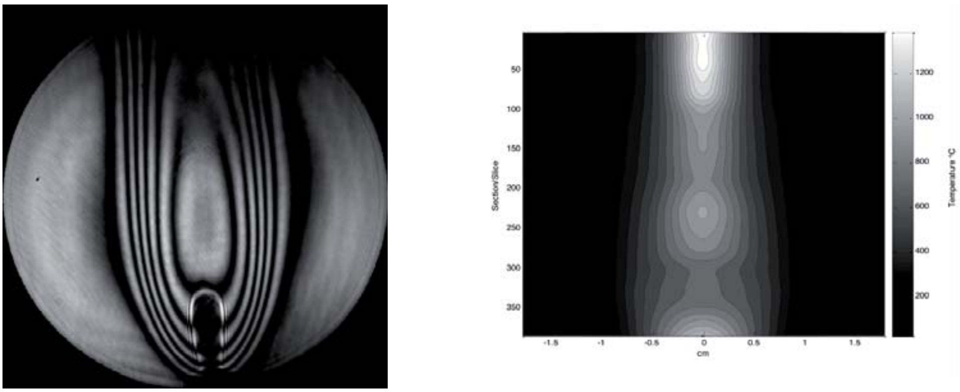
Figure 7 shows an interferogram obtained with a PDI and a 532.8 nm-wavelength laser. A simple rotation is applied to the interferogram to see vertically. The temperature (T) is found by using the Gladstone-Dale relationship [7]

$$n-1=\frac{0.294036x10^{-3}}{1+0.369203x10^{-2}T} \tag{1.18}$$



a) *Non- constrained approximation* b) *Constrained approximation*

Figure 6: *Functions approximation, n_g = {5, 9, 13}, d =0 y s = 1.*



a) Candle flame interferogram *b) Temperature distribution*

Figure 7: *Interferogram and temperature distribution of a candle flame.*

Conclusion

We have shown a temperature estimation method for the optical tomographic reconstruction using only one projection of a smooth phase object. With this method we can obtain any longitudinal or cross-sectional section of the volumetric distribution of the temperature of a candle flame.

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