Regularized quadratic cost function for oriented fringe-pattern filtering

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We use the regularization theory in a Bayesian framework to derive a quadratic cost function for denoising fringe patterns. As prior constraints for the regularization problem, we propose a Markov random field model that includes information about the fringe orientation. In our cost function the regularization term imposes constraints to the solution (i.e., the filtered image) to be smooth only along the fringe's tangent direction. In this way as the fringe information and noise are conveniently separated in the frequency space, our technique avoids blurring the fringes. The attractiveness of the proposed filtering method is that the minimization of the cost function can be easily implemented using iterative methods. To show the performance of the proposed technique we present some results obtained by processing simulated and real fringe patterns. © 2009 Optical Society of America

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The demodulation of digital fringe patterns is widely used in optical tests such as electronic speckle pattern interferometry (ESPI), holographic interferometry, or moiré interferometry. Several techniques can be applied for the extraction of the phase field; however, in the process of formation and acquisition of fringe patterns, noise commonly contaminates images. For this reason denoising fringe patterns plays an important role to make phase extraction easier, more robust, and more accurate. However, the frequencies of fringes and noise usually overlap and normally cannot be separated properly, and common filters for image processing have blurring effects on fringe features, especially for patterns with high-density fringes. For these cases the use of anisotropic filters is a better way for removing noise without blurring effects.

In the field of image processing, the regularization theory [1–3] has been demonstrated to be a powerful tool for reconstructing images. Particularly, in the past few years some works have been developed for fringe analysis, among them are the works in [4,5]. Although directional filtering has been studied for fringe images, for example, the outstanding work by Tang *et al.* [6] that proposed second-order oriented partial-differential equations for denoising ESPI fringes, we use a different and powerful mathematical tool for this purpose. In this Letter we derive a regularized quadratic cost function that is used for denoising along fringes in this kind of images.

It is widely known that the problem of reconstructing an image x from a degraded image y, i.e., the observed image, is often formulated according to the model

$$y = H(x) + n, \tag{1}$$

where n is the additive noise and H may represent a linear operator that is assumed to be known, which may be some kind of distortion. For example, H may be the point spread function of the imaging system. In general, the information provided by the observations of y is not enough for a proper estimation of x. For an adequate recovery of x we need to regularize the problem including prior information about the characteristics of the field to be estimated. The stochastic route to regularize the problem described in Eq. (1) may be derived using the Bayesian estimation. Using the Bayes's rule, one may model the posterior distribution of x with a given y as

$$P_{x/y}(x) = K P_{y/x}(x) P_x(x), \tag{2}$$

where K is a constant and

$$P_{y/x}(x) = K_1 \exp\left[- \sum_{m \in L} \Phi(H(x_m) - y_m) \right] \tag{3}$$

represents the conditional distribution of y with a given x, Φ is a potential function that is defined by the noise model, $m=(i,j)$ is the image coordinates in a regular lattice L, and K_1 is a constant. The prior distribution $P_x(x)$ that is commonly used in the framework of the Bayesian regularization are the Markov random fields (MRFs) [3,7,8], which are defined by a set C of potential functions V_c that ranges over the cliques associated with a given neighborhood system. An important characteristic of the MRFs is that the probabilistic dependencies of the elements of the estimated field are local, which make MRFs adequate for modeling piecewise smooth functions. Using an MRF, $P_x(x)$ is then given by the Gibbs distribution

$$P_x(x) = K_2 \exp\left[- \sum_{C} V_c(x_m) \right], \tag{4}$$

where K_2 is a normalizing constant.

Then, we can define the maximum *a posteriori* estimator as the minimizer of a cost function of the form

$$U(x) = \sum_{m \in L} \Phi(H(x_m) - y_m) + \mu \sum_{C} V_c(x_m), \qquad (5)$$

where parameter μ is a parameter that depends on the noise variance. It is a positive number that controls the compromise between the degree of regularization and its closeness with the observed data.

Finally, the regularized solution $\hat{x}$ is given by

$$\hat{x} = \underset{x}{\operatorname{argmin}}[U(x)]. \qquad (6)$$

The first term in Eq. (5), called the data term, establishes that the reconstruction of x must be consistent with the observed data. The second term in Eq. (5) is the well-known regularization term that imposes a penalty for violating the *a priori* assumptions. For example, considering a Gaussian noise and an MRF with a neighborhood system C that corresponds to the horizontal, vertical, and diagonal first-neighbor elements, that is, $C=\{l \in L: 0<\|m-l\| \leq \sqrt{2}\}$, the regularized cost function may be of the form

$$U(x_m) = \sum_{m \in L} [x_m - y_m]^2 + \mu \sum_{\langle m,l \rangle \in L} [x_m - x_l]^2. \qquad (7)$$

In this equation the first term imposes the estimated field to be close to the data in the sense of least squares. On the other hand, the second term imposes the estimated field to minimize the square differences along the horizontal, vertical, and diagonal directions. This means that the estimated image $\hat{x}$ by minimizing this cost function represents a simple low-pass filtered image. Unfortunately, the model of cost function (7) represents an isotropic filter that smoothes the original image in all directions, which may blur fringe information.

To derive a cost function for filtering along the fringe's tangent direction, we now consider the field of orientations θ_m. The discrete version of the partial derivatives along the fringes can be defined as

$$\frac{\partial x}{\partial \rho} = (x_m - x_h)\cos\,\theta_m + (x_m - x_v)\sin\,\theta_m, \qquad (8)$$

where h and v represent the first neighbors of m along the horizontal and vertical directions, respectively. Now, we propose the following cost function:

$$U(x_m) = \sum_{m \in L} [x_m - y_m]^2 + \mu \sum_{\langle m,h,v \rangle \in L} [(x_m - x_h)\cos\,\theta_m + (x_m - x_v)\sin\,\theta_m]^2. \qquad (9)$$

The minimization of this cost function is equivalent to smoothing the image only along the fringe's tangent direction.

Introducing the image coordinates i,j we can rewrite Eq. (9) in the following way:

$$U(x_{i,j}) = \sum_{(i,j) \in L} \left\{ [x_{i,j} - y_{i,j}]^2 + \mu \left[\left(\frac{\partial x}{\partial \rho}\right)^2_{i,j} + \left(\frac{\partial x}{\partial \rho}\right)^2_{i+1,j} + \left(\frac{\partial x}{\partial \rho}\right)^2_{i,j+1} \right] \right\}, \qquad (10)$$

where

$$\left(\frac{\partial x}{\partial \rho}\right)_{i,j} = (x_{i,j} - x_{i-1,j})c_{i,j} + (x_{i,j} - x_{i,j-1})s_{i,j},$$

$$\left(\frac{\partial x}{\partial \rho}\right)_{i+1,j} = (x_{i+1,j} - x_{i,j})c_{i+1,j} + (x_{i+1,j} - x_{i+1,j-1})s_{i+1,j},$$

$$\left(\frac{\partial x}{\partial \rho}\right)_{i,j+1} = (x_{i,j+1} - x_{i-1,j+1})c_{i,j+1} + (x_{i,j+1} - x_{i,j})s_{i,j+1}, \qquad (11)$$

$c_{i,j}=\cos\,\theta_{i,j}$ and $s_{i,j}=\sin\,\theta_{i,j}$.

To minimize cost function (10) we have to solve the following linear system, which is obtained by setting the partial derivative of $U(x)$ with respect to $x_{i,j}$ and equating it to 0:

$$\frac{\partial U}{\partial x_{i,j}} = 2[x_{i,j} - y_{i,j}] + 2\mu[(x_{i,j} - x_{i-1,j})c_{i,j} + (x_{i,j} - x_{i,j-1})s_{i,j}](c_{i,j} + s_{i,j}) - 2\mu[(x_{i+1,j} - x_{i,j})c_{i+1,j} + (x_{i+1,j} - x_{i+1,j-1})s_{i+1,j}]c_{i+1,j} - 2\mu[(x_{i,j+1} - x_{i-1,j+1})c_{i,j+1} + (x_{i,j+1} - x_{i,j})s_{i,j+1}]s_{i,j+1} = 0. \qquad (12)$$

Once the orientation field θ_m is previously estimated, the field $x_{i,j}$ can be computed using iterative methods [9]. In our experiments we used the simple gradient-descent algorithm, which is described by the iterations

$$x^{k+1} = x^k - \lambda \frac{\partial U^k}{\partial x}, \qquad (13)$$

where λ is a positive constant.

The implementation of the filtering method requires the previous computation of the field θ at each cite in the image. For this purpose we use the technique reported by Yang *et al.* [10]. We briefly describe this method for the computation of fringe orientation. The authors computed the fields of image differences in the horizontal, vertical, and diagonal directions,

$$d^0_{i,j} = |y_{i-1,j} - y_{i+1,j}|\sqrt{2}, \quad d^{45}_{i,j} = |y_{i-1,j+1} - y_{i+1,j-1}|,$$

$$d^{90}_{i,j} = |y_{i,j-1} - y_{i,j+1}|\sqrt{2}, \quad d^{135}_{i,j} = |y_{i-1,j-1} - y_{i+1,j+1}|. \qquad (14)$$

Owing to the computation of central differences in diagonal directions, the factor $\sqrt{2}$ is used in the horizontal and vertical differences for consistency. For es-

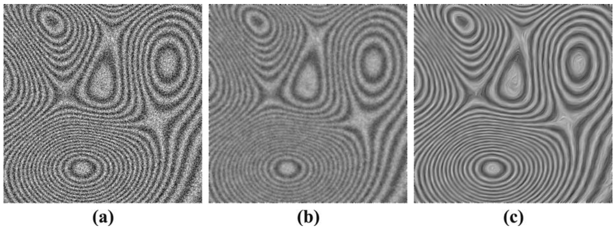

Fig. 1. (a) Simulated noisy fringe pattern. (b) Result obtained by minimizing cost function (7). (c) Result obtained by minimizing cost function (10).

timating the angle β along the fringe's orthogonal direction, they consider square regions $\Gamma \in L$ with center (i,j) such that

$$\beta_{i,j} = \frac{1}{2}\arctan\left(\sum_{(k,l)\in\Gamma} d_{k,l}^{45} - \sum_{(k,l)\in\Gamma} d_{k,l}^{135}, \sum_{(k,l)\in\Gamma} d_{k,l}^{0} - \sum_{(k,l)\in\Gamma} d_{k,l}^{90} \right), \tag{15}$$

where the function $\arctan(\cdot,\cdot)$ returns values in the interval $[-\pi,\pi)$, so that $\beta \in [-\pi/2, \pi/2)$. Finally, we can compute the field θ just by knowing that $\theta = \beta \pm \pi/2$.

We show the performance of the filtering method with the following three experiments. In all the experiments we used the value $\lambda = 0.01$ with 30 iterations for the optimization algorithm in a 2.66 GHz Pentium D based computer using MATLAB.

Figure 1(a) is a simulated noisy fringe pattern of size 400×400. In Figs. 1(b) and 1(c) we show the results obtained by minimizing cost functions (7) and (10), respectively. We used the value $\mu = 4$ in both cases. Optimizing cost function (10) required 3.1 s. Figure 2(a) is a real moiré fringe pattern of size 220×220. In Figs. 2(b) and 2(c) we show the results obtained by minimizing cost functions (7) and (10), respectively. In this case we used the values $\mu = 3$ and $\mu = 10$, respectively. Optimizing cost function (10) required 0.9 s. Figure 3(a) is an experimentally obtained ESPI fringe pattern of size 200×200. Using cost function (7) with $\mu = 5$ we obtained the result shown in Fig. 3(b). Using the oriented filtering method with $\mu = 20$ we obtained the result shown in Fig. 3(c). Optimizing cost function (10) required 0.7 s.

The results show that filtering the images by means of cost function (7), the fringes are blurred specially in high-frequency zones. On the other hand, using our proposed method, the filtering operation preserves the fringes even in high-frequency zones.

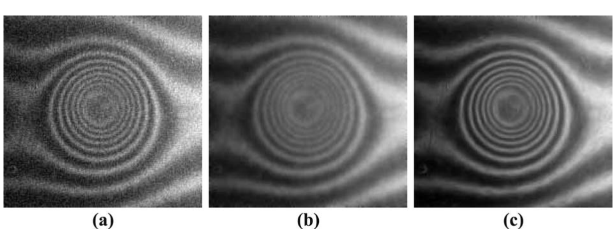

Fig. 2. (a) Real moiré fringe pattern. (b) Result obtained by minimizing cost function (7). (c) Result obtained by minimizing cost function (10).

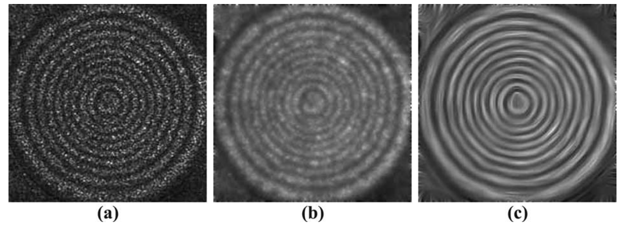

Fig. 3. (a) Real ESPI fringe pattern. (b) Result obtained by minimizing cost function (7). (c) Result obtained by minimizing cost function (10).

We can observe that in both cost functions, the parameter μ controls the wide band of the filter (the higher the value of μ the narrower the wide band); however, in cases of high noise levels increasing the value of μ using cost function (7) may eliminate fringe information in high-frequency zones, as experiments show. On the other hand, with cost function (10) we can increase the value of μ preserving the fringe information because the filtering operation is realized only along the fringe's tangent direction. The key factor of this characteristic is the proposed MRF, which penalizes big directional derivatives along the fringes by including orientation information. In this way the proposed filtering method may be considered adaptive in the sense that it adapts its filtering direction at each pixel in the image.

In some cases the filtering method presented here has a low performance in low-frequency zones. This problem, however, is not owing to cost function (10) itself but to the algorithm for estimating the fringe orientation. Unfortunately, as far as we know, all algorithms for estimating fringe orientation are so sensitive to low modulation and noise in very low-frequency zones. In particular, with the technique adopted in our work [10], this problem may be reduced by increasing the size of region Γ [Eq. (15)]; however, this may affect the estimation in high-frequency zones.

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