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# Robust profilometer for the measurement of 3-D object shapes based on a regularized phase tracker

J. Villa\*, M. Servin

Centro de Investigaciones en Optica, Apdo. postal 1-948, Leon, Guanajuato 37000, Mexico Received 22 April 1999; received in revised form 2 June 1999; accepted 2 June 1999

### Abstract

A robust profilometer is proposed to measure 3-D object shapes based on a regularized phase tracker that is capable of demodulating fringe patterns with a high noise presence and broad bandwidth due to the object shape. As shown herein, the technique acts as an adaptive filter and is capable of giving the detected phase continuously so that no further unwrapping process is required. Experimental results of real surface profiles are presented. © 1999 Published by Elsevier Science Ltd. All rights reserved.

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# 1. Introduction

Measurements of 3-D object shapes based on optical systems have wide application in many fields because of its high accuracy, noncontact, and automatic processing. An easy way to characterize and measure the 3-D information is by projecting a grating over the surface under test. This projected grating is observed by a camera. The observed pattern is phase-modulated due to the topography of the object, thus, depth information may be retrieved from the pattern by a demodulation process.

Fourier techniques [1–6] are some of the best methods for the demodulation process but they commonly fail with wideband noisy grating patterns modulated by broad bandwidth signals. On the other hand, if we use spatial synchronous methods based on window filters [7], we could have similar problems or ringing effects. Overall, the techniques mentioned above are based on different kinds of linear

<sup>\*</sup> Corresponding author.

E-mail address: jvilla@andromeda.cio.mx (J. Villa)

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filtering which requires prior information about the data to be processed (i.e., statistics of noise and bandwidth of the modulating signal).

Since the choice of the filter's parameters depend on the practical problem, it is not always easy to automatically process the fringe patterns. This kind of filtering may be effective with narrow bandwidth signals, however, it could fail with a broad bandwidth and high noise presence.

There exists another kind of technique to measure the depth information contained in grating patterns based on the phase-stepping interferometry [8]. This technique requires at least three phase-shifted grating patterns, however, the phase-shifting arrangement is complicated and is much more sensitive to noise and nonlinearities.

Most of the methods to demodulate fringe patterns included those mentioned above give the detected phase wrapped because they use the arctan function so that an unwrapping process is required to retrieve the 3-D information. In some applications in which the presence of noise is low the unwrapping could be easy but in most of the cases we may require robust unwrapping methods which implies a computational intensive task. Evenmore, we could have inconsistences in the wrapped phase that might introduce errors even with robust unwrapping methods [9].

In this paper it is demonstrated that the use of a phase-tracker profilometer (PTP) [10] can alleviate the problems mentioned above because it acts as an adaptive filter and requires just a single grating pattern to retrieve the 3-D information in an automatic manner. Furthermore, the technique gives the detected phase continuously so that no further unwrapping process is required.

## 2. Optical geometry for height measurement

A typical crossed-optical-axes geometry of the projection and recording system for the height measurement is shown in Fig. 1. The optical axes of the projector and the camera crosses at point O on an imaginary plane R that serves as a reference from which the object height is measured.  $E_p$  and  $E_c$  denote the projection centers of the projector and the CCD camera, respectively. L is an imaginary plane on which the image of the grating is formed. The distance between  $E_p$  and  $E_c$  is denoted by d, and  $l_o$  is the distance between  $E_c$  and O. P is the period of the projected grating. The lines of the grating are normal to the plane of the figure.

If the grating is defocusedly projected onto the object [3], the image observed through the CCD camera will have a quasi-sine distribution which can be written as

$$g(x, y) = a(x, y) + b(x, y)\cos[2\pi f_0 x + \phi(x, y)],$$
(1)

$$\phi(x, y) = \phi_0(x, y) + \phi_z(x, y),$$
(2)

where  $f_0$  is the fundamental frequency of the observed grating image, a(x, y) and b(x, y) are the background and the amplitude modulation, respectively,  $\phi_0(x, y)$  is the phase modulation by diverging illumination in z(x, y) = 0, and  $\phi_z(x, y)$  is the phase shift due



Fig. 1. Crossed-optical-axes geometry of the projection and recording system for the height measurement.

to the object's height distribution, thus

$$\phi_z(x, y) = \phi(x, y) - \phi_0(x, y) = 2\pi f_0 CD.$$
(3)

The formula to obtain the height distribution is [1]

$$z(x, y) = \frac{l_o CD}{CD - d} = \frac{l_o \phi_z(x, y)}{\phi_z(x, y) - 2\pi f_o d}.$$
(4)

#### 3. The filtering problem for phase measurement

We will briefly analyze the filtering problem to recover the phase from fringe patterns modulated by 3-D information. As mentioned above, synchronous and Fourier methods are based on linear filtering. The filters are applied to a set of noisy data (i.e., the fringe pattern) in order to extract information about a prescribed quantity of interest (phase).

The noise may arise from a variety of sources such as noisy sensors, specklelike structures, and shadows in the observation process. From the statistical viewpoint for the solution of the linear filtering problem, we assume availability of certain parameters of the useful signal and unwanted noise. In conventional signal processing we also assume that the degrading signal (noise) is spatially invariant. Since the choice of the filer is related to the practical problem, it is not always straightforward to automatically process and recover the phase.

A more efficient way to extract the information of interest from a signal is to use an adaptive filter. By such a device we mean a self-designing, which makes possible for the filter to perform satisfactorily in an environment where a complete knowledge of the relevant signal characteristics is not available. The principal characteristic of an adaptive filter is that its parameters become data dependent. This, therefore, means that an adaptive filter is in reality a nonlinear device because it does not obey the principle of superposition.

## 4. Review of the regularized phase-tracking technique

As mentioned above, to recover the 3-D information from the fringe-grating pattern we have to extract the phase information of a signal modeled by Eq. (1). The phase detection is an inverse problem that must be solved by an algorithm that incorporates smoothness constraints about the phase being detected. To solve this problem we may regularize it by proposing a suitable cost function with a term related to the fidelity between the estimated phase function and the observation, and other term related to the smoothness of the phase being estimated. It is then assumed that the estimated phase function is the minimizer of the proposed cost function.

In the RPT technique the local irradiance is modeled as a cosinusoidal function phase-modulated by a plane, that is because it is assumed that locally the fringe pattern is considered as spatially monochromatic. The modeled cosinusoidal function must be close to the irradiance of the pattern which corresponds to the fidelity term. In this way, the phase plane must be adapted to every region in the pattern. The smoothness and the continuity of the estimated phase are enforced by the second term of the cost function which is expressed as

$$U_{x,y}(\phi, \omega_x) = \sum_{(\tilde{x}, \tilde{y})\in (N_{x,y} \cap L)} [|g_h(\tilde{x}, \tilde{y}) - \cos[\phi_e(x, y, \tilde{x}, \tilde{y}) + \omega_0 \tilde{x}]|^2 + \lambda |\phi(\tilde{x}, \tilde{y}) - \phi_e(x, y, \tilde{x}, \tilde{y})|^2 m(\tilde{x}, \tilde{y})],$$
(5)

where

$$\phi_e(x, y, \tilde{x}, \tilde{y}) = \phi(x, y) + \omega_x(x, y)(x - \tilde{x}) + \omega_y(x, y)(y - \tilde{y}), \tag{6}$$

where  $U_{x,y}$  is the energy of the system at a site (x, y) in the image. L is a twodimensional lattice that has valid data and  $N_{x,y}$  is a neighborhood region around the coordinate (x, y) where the phase is being detected. The field m(x, y) is an indicator that equals 1 if the site has already been estimated and 0 otherwise.  $\omega_x$  and  $\omega_y$  are the estimated local frequencies in the x and y directions, and  $\omega_o$  is the carrier frequency.  $\lambda$  is a regularizing parameter that controls the smoothness of the detected phase. In order to eliminate the background a(x, y) of Eq. (1), g(x, y) is replaced by a highfiltered version  $g_h(x, y)$ . As can be seen, the RPT system represented by Eq. (5) attempts to adapt every local phase plane to the observed data and to the phase values that have already been estimated (self-designing and data dependence which are characteristics of an adaptive filter).

#### 5. The crystal-growing algorithm for the phase estimation

To demodulate the fringe pattern, we have to use a sequential algorithm to minimize  $U_{x,y}$  at each site with respect to  $(\phi, \omega_x, \omega_y)$ . The algorithm starts by setting the indicator *m* to zero. We choose a starting point  $(x_0, y_0)$  inside *L*. The cost function  $U_{x,y}$  is then minimized with respect to  $\phi(x_0, y_0), \omega_x(x_0, y_0), \omega_y(x_0, y_0)$  and we set  $m(x_0, y_0) = 1$ . The sequential phase-demodulation algorithm then will proceed as follows:

(1) Choose an (x, y) pixel inside L (may be with a prescribed scanning order).

(2) (a) If no adjacent pixel has already been estimated, or m(x, y) = 1, return to step (1).

(b) Take the value of  $(\phi, \omega_x, \omega_y)$  of an adjacent pixel as the initial condition to minimize  $U_{x,y}$ .

(3) Set m(x, y) = 1.

(4) Repeat the process until all the sites in L are estimated.

To minimize  $U_{x,y}$  with respect to  $(\phi, \omega_x, \omega_y)$  at each site we may use the gradient descent algorithm:

$$\phi^{k+l}(x, y) = \phi^{k}(x, y) - \mu \frac{\partial U_{x,y}(\phi, \omega_{x}, \omega_{y})}{\partial \phi(x, y)},$$

$$\omega^{k+l}_{x}(x, y) = \omega^{k}_{x}(x, y) - \mu \frac{\partial U_{x,y}(\phi, \omega_{x}, \omega_{y})}{\partial \omega_{x}(x, y)},$$

$$\omega^{k+l}_{y}(x, y) = \omega^{k}_{y}(x, y) - \mu \frac{\partial U_{x,y}(\phi, \omega_{x}, \omega_{y})}{\partial \omega_{y}(x, y)},$$
(7)

where k is the iteration number and  $\mu$  is the step size that determines the stability and convergence rate which is typically about 0.01.

#### 6. Experimental results

A 100 lin/in Ronchi grating was used to be projected by a conventional slide projector. The deformed grating pattern was observed by a sony handicam video camera and stored to be processed into a 200 MHz Pentium computer with a frame memory of  $128 \times 128$  pixels (8-bits gray levels). The parameters in the optical arrangement shown in Fig. 1 were d = 40 cm and  $l_0 = 125$  cm. The period in the pattern image was  $p_0 = 5.44$  pixels and the period of the grating projected onto the object p = 2.2 mm.

A craftsmanship was used as an object under test in the first experiment. Fig. 2(a) shows the surface of the object. Fig. 2(b) is the observed fringe pattern. The best wrapped phase we could obtain by using the 2-D Fourier method with a Gaussian filter is shown in gray levels in Fig. 2(c).

Fig. 2(d) shows the phase obtained by using the PTP (wrapped for comparison purposes) with a  $N_{x,y} = 7 \times 7$  neighborhood region. The mesh of the object's height distribution by using the PTP is shown in Fig. 3.



Fig. 2. (a) Surface of the object under test in the first experiment. (b) Projected grating onto the craftsmanship. (c) The best wrapped phase obtained from image shown in Fig. 2(b) using the 2-D Fourier method. (d) Phase obtained from image shown in Fig. 2(b) using the PTP with a  $7 \times 7$  neighborhood region (shown wrapped for comparison purposes).

The face of a manikin with irregular reflectance was used in the second experiment. Fig. 4(a) shows the image of the grating projected onto the object. Fig. 4(b) shows the wrapped phase obtained using the 2-D Fourier method with the frequency sufficiently filtered to let the phase information through. As can be seen, phase information has been corrupted by noise so that a more narrow filter should be used to reject it. Fig. 4(c) shows the wrapped phase also obtained using the 2-D Fourier method but with a sufficiently narrow filter to reject the noise. In this case, the field has phase inconsistences in the nose because the filter has eliminated phase information. Fig. 4(d) shows the phase field (also wrapped) in gray levels using the PTP. The mesh of the object's height distribution by using the PTP is shown in Fig. 5.



Fig. 3. Height distribution obtained from the craftsmanship using the PTP.



Fig. 4. (a) Grating projected onto a face of a manikin with irregular reflectance. (b) Wrapped phase obtained from the image shown in Fig. 4(a) using the 2-D Fourier method with the frequency sufficiently filtered to let the information trough. (c) Wrapped phase obtained from image shown in Fig. 4(a) using the 2-D Fourier method with an enoughly narrow filter to reject the noise. (d) Phase obtained from image shown in Fig. 4(a) (shown wrapped for comparison purposes) using the PTP.



Fig. 5. Height distribution of the face using the PTP.



Fig. 6. Height distribution of the face using the 2-D Fourier method and the unwrapping method proposed by Ghiglia et al.

The experiments show that by using the PTP we can get a better separation of the height information from noise because the characteristics of the PTP attempts to adapt every local phase plane to the observed data and to the phase values that have already been estimated which means that it acts as an adaptive filter. On the other hand, since the Fourier method is based on linear filtering systems we cannot always choose adequate filters to reject the noise without eliminating the phase information and it is not always easy to process the fringe pattern because the choice of the filter is related to the practical problem.



Fig. 7. (a) A profile of the height distribution shown in Fig. 5. (b) A profile of the height distribution shown in Fig. 6.

As mentioned above, the phase inconsistences introduced by linear filtering might introduce errors in the unwrapped phase even if we use a robust unwrapping method. Fig. 6 shows the height distribution applying the unwrapping method proposed by Ghiglia et al. [9] in the wrapped phase shown in Fig. 4(c). Fig. 7(a) shows a profile of the height distribution shown in Fig. 5. Fig. 7(b) shows a profile of the height distribution shown in Fig. 6.

## 7. Conclusions

The use of linear systems such as Fourier methods to retrieve the 3-D information may be limited by the choice of filters depending on the practical problem, furthermore, they may also be restricted when we have noisy broad bandwidth patterns due to a compromise between the phase retrieval and the noise rejection. In order to avoid these kinds of restrictions we have proposed the use of the PTP which has characteristics of an adaptive filter and can process the grating patterns in a more automatic manner. Another advantage of the PTP is that it gives the phase already unwrapped.

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